國立中央大學106學年度碩士班考試入學試題

所別： 數學系 碩士班 應用數學組(一般生)　　共1頁　第1頁

科目： 微積分

本科考試禁用計算器　　須有計算過程　　*請在答案卷內作答

參考用

Problem 1. (12%) Find $\dfrac{d}{dx}\displaystyle\int_{2^x}^{\cos^{-1}x} \ln(u^4+1)\,du$.

Problem 2. Let $I_n = \displaystyle\int_0^{\frac{\pi}{2}} \sin^n x\,dx$.

(a) (15%) Show that $I_{2k} = \dfrac{(2k)!}{(2^k k!)^2}\dfrac{\pi}{2}$ and $I_{2k+1} = \dfrac{(2^k k!)^2}{(2k+1)!}$ for all $k \in \mathbb{N} \cup \{0\}$.

(b) (5%) Show that $\displaystyle\lim_{k\to\infty} \frac{I_{2k+1}}{I_{2k}} = 1$. $\left(\textbf{Hint}\text{: show and use the fact that } \dfrac{I_{2k+2}}{I_{2k}} \leqslant \dfrac{I_{2k+1}}{I_{2k}} \leqslant 1\right)$

(c) (8%) Use (b) to find the limit $\displaystyle\lim_{k\to\infty} \frac{k!e^k}{k^{k+0.5}}$.

Problem 3. (15%) Find the indefinite integral $\displaystyle\int \frac{1}{2+\cos x}\,dx$.

Problem 4. (10%) Find the Maclaurin series of the the function $f(x) = \tan^{-1}(x^2)$. Note that you need to specify the interval of convergence of the Maclaurin series.

Problem 5. (13%) Find all $p \in \mathbb{R}$ such that the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(\ln n)^p}{n}$ is convergent.

Problem 6. (12%) Find all relative extrema and saddle points of the function $F(x,y) = 2xy - \frac{1}{2}(x^4+y^4)+1$.

Problem 7. (10%) Let $w = f(x,y,z)$ and $g(r,\theta,z) = (r\cos\theta, r\sin\theta, z)$. Show that

$$(w_x^2+w_y^2+w_z^2)\circ g = \left|\frac{\partial(w\circ g)}{\partial r}\right|^2 + \frac{1}{r}\left|\frac{\partial(w\circ g)}{\partial \theta}\right|^2 + \left|\frac{\partial(w\circ g)}{\partial z}\right|^2,$$

where w_x, w_y, w_z denote the partial derivative of w with respect to x, y, z, respectively.