所別： 機械工程學系 碩士班 系統組(一般生)　　　　　　　共之頁 第1頁

科目： 工程數學(含程式設計)

本科考試可使用計算器，廠牌、功能不拘　　須有計算過程　　＊請在答案卷　內作答

1. Solutions for ordinary differential equations (ODEs) (25%)

(a) Find the solution for the ODE $e^{3\theta}(dr + 3rd\theta) = 0$　　　　　　　　**(5%)**

(b) Find the solution for the ODE $y'' + 16y = 4\sin t$, $y(0) = 0$, $y'(0) = 1$　　**(10%)**

(c) Find a basis of solutions by the Frobenius method of the following ODE: $x^2 y'' + x(2x - 1)y' + (x + 1)y = 0$.　　　　　　　　　　**(10%)**

2. Series solution and Linear algebra (25%)

(a) For a general **Legendre** equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ Please derive the recursion relation. **(7%)** and use power series method to solve it as n=1. **(8%)**

(b) Please use **Cramer's rule** to evaluate A_n and B_n of the following equations **(10%)**

$$\begin{cases} (25 - n^2)A_n + 0.05nB_n = \dfrac{4}{n^2\pi} \\ -0.05nA_n + (25 - n^2)B_n = 0 \end{cases}$$

3. Laplace transform / Fourier analysis (25%)

(1) Fourier series expansion of a periodic function (or signal) $f(t)$ can be represented as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nw_0 t + \sum_{n=1}^{\infty} b_n \sin nw_0 t,$$

where $a_0 = \frac{2}{T}\int_T f(t)dt$, $a_n = \frac{2}{T}\int_T f(t)\cos n\omega_0 t\,dt$, and $b_n = \frac{2}{T}\int_T f(t)\sin n\omega_0 t\,dt$.

(i) **(6%)** Give the physical meaning of $a_0/2$ and ω_0.

(ii) **(8%)** If now $g_1(t)$ is an **even** function, please address and rewrite its Fourier series expansion; likewise, if $g_2(t)$ is an **odd** function, what does the form of its Fourier series expansion become?

(2) If a measured temperature can be characterized as a square wave of amplitude $20\,°C$ and period $T = 6\,sec$,

(i) **(3%)** please first **sketch the time waveform of measured temperature** stated as the above in the form of an **even function _or_ an odd function** (Time in second: $-\infty \leftrightarrow \infty$);

(ii) **(8%)** then, derive its Fourier series expansion (till **the first six terms**).

注意：背面有試題

4. Programming (25%)

(a)(10%) A right triangle can have sides that are all integers. A set of three integer values for the sides of a right triangle is called a Pythagorean triple. These three sides must satisfy the relationship that the sum of the squares of two of the sides is equal to the square of the hypotenuse. Write a program that shows all Pythagorean triples for side1, side2 and hypotenuse all no larger than 500. Use a **triple-nested loop** that tries all possibilities. The code is limit to **C, C++, Visual Basic** or **Fortran** programing language, and please state before your answer. All variables declare to integers.

(b)(15%) Write a program that estimates the value of the mathematical constant e by using the formula:

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$$

Using this formula, write a **function** that accepts the value of n as the number of terms in the summation from a calling function. The return value of this function should be the value of e. The code is limit to **C, C++, Visual Basic** or **Fortran** programing language, and please state before your answer. The variables declare to real numbers except for n as integer.