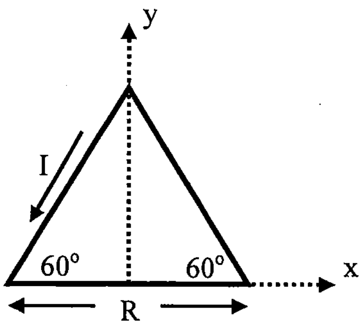
國立中央大學 107 學年度碩士班考試入學試題

所別： 光電科學與工程學系 碩士班 不分組(一般生) 共2頁 第1頁

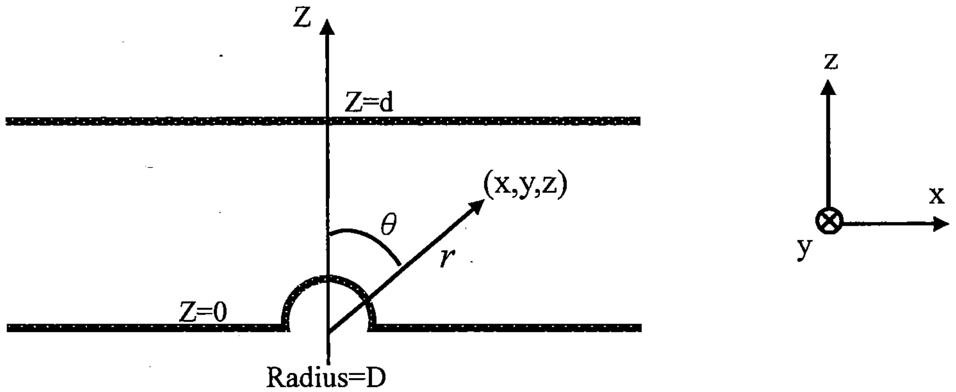
科目： 電磁學

本科考試可使用計算器，廠牌、功能不拘 *請在答案卷(卡)內作答

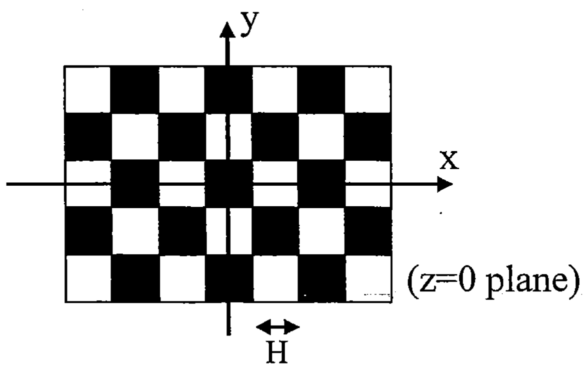
1. A filamentary conductor is formed into an equilateral triangle with sides of length R carrying current I. Find the magnetic field intensity at the center of the triangle. (14%)



2. A large, charged, flat metal plate with a hemispherical protrusion, of radius D, is placed on the x-y plane. An equally and oppositely charged (surface charge density $-\sigma_o$) plane conductor is situated on the plane z=d, as shown in the figure below, where D<<d. Find the electric field in the space between two plates and the distribution of charge on the plate with the protrusion. (18%)



3. An infinite checkerboard in the x-y plane are supplied with the potential distribution such that the black squares are at potential $+V_0$ and the white squares are at potential $-V_0$, as shown in the figure below. The width and the height of each square is H. The plane parallel to and above this plane at z=L is maintained at zero potential. Find the potential at all points between z=0 plane and z=L plane. (18%)



注意:背面有試題

參考用

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4. An equilateral triangle loop of wire, with sides of length R, lies in the second quadrant of xy plane, having a nonuniform time-dependent magnetic field, $\vec{B}(x,y,t) = kxy^2t^3\hat{z}$, where k is a constant. Please find the emf induced in the loop (10%).

5. An infinitely long cylindrical tube of radius a moves at constant speed v along its axis. It carries a net charge per unit length λ, uniformly distributed over its surface. Surrounding it, at radius b, is another cylinder, moving with the same velocity but carrying the opposite charge ($-\lambda$). (a) Please explain what the Poynting vector, $\vec{S}$, is (4%). (b) Find the power transported by the fields across a plane perpendicular to the cylinders (6%).

6. (a) Calculate the light reflection (%) from a boundary of air and glass if the light is incident normally from air into the glass ($n_{glass} = 1.5$) (5%). (b) Regarding the reflection (R) and transmission (T) at oblique incidence, if the incoming light meets the boundary (refractive indexes, n_1 and n_2, are 1 and $\sqrt{3}$, respectively) at an incident angle of 60°, calculate the R and T (5%).

7. Regarding the electromagnetic waves in conductors, if the Maxwell's equations and plane-wave solutions are given as follows,

Maxwell's equations: (i) $\nabla \cdot \vec{E} = 0$; (ii) $\nabla \cdot \vec{B} = 0$; (iii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$; (iv) $\nabla \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t} + \mu\sigma\vec{E}$

Plane-wave solutions: $\vec{\tilde{E}} = \vec{\tilde{E}}_0 e^{i(\tilde{k}z-\omega t)}$; $\vec{\tilde{B}} = \vec{\tilde{B}}_0 e^{i(\tilde{k}z-\omega t)}$, where $\tilde{k}$, σ, μ, and ε are complex wave number ($k+i\kappa$), conductivity, permeability and permittivity, respectively.

(a) Derive the skin depth (d) in terms of ω, ε, μ, and σ; also describe its physical meaning (10%).

(b) Derive the real amplitude ratio of B_0/E_0 in terms of ω, ε, μ, and σ, and describe the physical meaning of a complex wave number (10%).

The following formulas may help.

◆The components of EM wave in waveguides for E_x, E_y, B_x, and B_y, as well as the uncoupled equations.

$$E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right), \qquad E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right), \qquad B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{\omega}{c}\right)^2 - k^2\right]\begin{matrix}E_z\\ B_z\end{matrix} = 0, \text{uncoupled equations}$$

where ω, c, and k are angular frequency, light velocity, and wave number, respectively

◆Fresnel's equations for the case of the polarization of the incident wave in the plane of incidence.

$$\tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}; \quad \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I} \qquad \alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}; \quad \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2}$$

where θ_I, θ_T, μ, v, and the suffixes of "1" and "2", denote angle of incidence, angle of refraction, permeability, light velocity in media, medium for incidence, and medium for transmission, respectively.

參考用

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