國立中央大學107學年度碩士班考試入學試題

所別： 資工類　　　　共6頁　第1頁

科目： 離散數學與線性代數

本科考試禁用計算器　　　　*請在答案卷(卡)內作答

多重選擇題 (每題5分、每一選擇答對給1分、答錯倒扣1分、不答0分)

I. 離散數學（50分）

1. Which of the following are correct ways to verify if two compound propositions *p* and *q* are logically equivalent?
   - A. Show that $p \leftrightarrow q$ is a tautology.
   - B. Show that $p \leftrightarrow q$ is a contradiction.
   - C. Show that *p* and *q* contain the same truth values as each other in some rows of their truth tables.
   - D. Use equivalence laws to derive *p* from *q*.
   - E. Use equivalence laws to derive *q* from *p*.
2. For two arbitrary infinitely countable sets *A* and *B*, which of the following statements are true.
   - A. Both 2^A and 2^B are uncountable.
   - B. $A - B$ can be ϕ.
   - C. $A - B$ can be finite.
   - D. $A - B$ can be infinitely countable.
   - E. $A - B$ can be uncountable.
3. Which of the following statements are correct?
   - A. If $|A| = r$ and $|B| = n$, there are n^r different functions from A to B.
   - B. There are $\binom{13}{3}$ possible non-negative integer solutions to the equation: $x_1 + x_2 + x_3 + x_4 = 10$.
   - C. There are $\binom{n}{r}$ bit strings of length *n* containing exactly *r* 1's.
   - D. There are 23 ways to distribute 10 items to 4 identical empty boxes.
   - E. There are $\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}$ ways to distribute hands of 13 cards to each of four players from the standard deck of 52 cards.
4. Which of the following statements are correct?
   - A. A pseudograph may contain edges that connect a node to itself.
   - B. The number of nodes for W_n is $n + 1$.
   - C. The number of edges for *n*-Cubes Q_n is $(n-1)2^n$.
   - D. The sum of the degree for all nodes in an undirected graph is even.
   - E. A path with length *n* in a simple graph of *n* nodes must contain a loop.
5. A poset with the relation R is represented by the Hasse diagram in Figure 1. We can conclude that:

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參考用

A. R is a total order.
B. R is not transitive.
C. R is reflexive.
D. The least upper bound of $\{a, c\}$ is d.
E. R is a lattice.

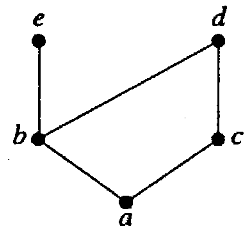

Figure 1. The Hasse diagram of R

6. Which of the following statements are true?
   A. If there exists an 1-to-1 mapping from set A to B, then $|A| = |B|$.
   B. If there exists an equivalent relation on a set, a partition on that set always can be formed.
   C. If mathematical induction can be applied on a set of predicates, these predicates can form a total-ordered set.
   D. $\forall n>0$, if number of length-n paths are all the same in graph G and H, then G is isomorphic to H.
   E. When proving f is $O(g)$, we must find the smallest c,k such that $\forall x>k$: $f(x) \le cg(x)$.

7. Analyze the time complexity of the following procedure P. Suppose P ,Q,and R are all procedures. Q will take $\sqrt{m}$ steps to process a length-m array into 4 length-$m/4$ arrays, $B_1, B_2, \ldots, B_4$; R will take $2\sqrt{m}$ steps to merge 2 size m arrays , where m is the size of input arrays. Each statement line in and outside the loop counts 1 step.

Procedure $P(\text{A}[a_1, a_2, \ldots, a_n])$

$B_1, B_2, \ldots B_4$ are initially empty arrays.

1. if n<4 exit.
2. call Q (A); /* and get $B_1, B_2, \ldots$ */
3. call $P(B_1)$;
4. call $P(B_4)$;
5. call R(B_1, B_4);
6. return;

Suppose n is a number of power of 4, What of the following options are true about the number of steps ($p(n)$) and complexity (C_p) of the procedure P in the question above? (C,,D are constants)

   A. $p(n) = 2p(n/4) + Cn^{1/2} + D$
   B. $p(n) = 4p(n/4) + Cn^{1/4} + D$
   C. $C_p = \theta(n \log n)$
   D. $C_p = \theta(\sqrt{n} \log n)$
   E. $C_p = \theta(\sqrt{n})$

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8. Existing 2 sets A and B, $|A| = 3, |B| = 5$, which of the following options are true?
   - A. the number of possible functions from A to B is 2^{15}.
   - B. the number of possible 1-to1 functions from A to B is 5^3.
   - C. the number of possible onto functions from A to B is 0.
   - D. the number of possible binary relations on $A \times B$ is 2^{15}.
   - E. the number of possible symmetric binary relations on $A \times B$ is 2^{10}.

9. Suppose x,y represents people. Let $S(x,y)$ be the predicate of "x is a senior of y," $F(x,y)$ be the predicate of "x is a friend of y," Choose the correct logic statement(s) which have the same meaning as the sentence ---**At least one of any person's senior is his friend**.
   - A. $\forall x (\forall y, F(y,x) \rightarrow S(y,x))$.
   - B. $\exists x(\forall y, \ (S(x,y) \wedge F(y,x))$.
   - C. $\forall x(\forall y, (S(y,x) \rightarrow F(x,y))$.
   - D. $\exists x(\forall y, F(y,x) \ \vee \ S(y,x))$.
   - E. none of the above.

10. What equations are true when using generating function to solve the recurrence series: $a_n = 6a_{n-1} - 9a_{n-2}, a_0 = 1, a_1 = 6$, which of the followings are true?
   - A. $g(z) = 1/(1-3z)^2$
   - B. $g(z) = 1/(1-3z^2)$
   - C. $g(z) = \frac{1+6z}{(1-3z)^2}$
   - D. $a_n = (7n+1)3^n$
   - E. $a_n = (n+1)3^n$

II. 線性代數（50分）

11. Concept of linear independent.
   - A. The linear system $\boldsymbol{Ax} = \boldsymbol{b}$ has unique solution for $\boldsymbol{b}$, then the columns of $\boldsymbol{A}$ are linearly independent.
   - B. The columns of the change-of-coordinate matrix $\boldsymbol{P}$ are linearly independent.
   - C. The subset of a linearly-dependent vector set is linearly dependent.
   - D. If $\boldsymbol{A}$ is diagonalization, then $\boldsymbol{A}$ has linearly-independent columns.
   - E. If $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}$ is invertible, then $\boldsymbol{A}$ has linearly-independent columns.

12. If $\boldsymbol{A}$ is diagonalizable and has eigenvalue λ, then
   - A. $\boldsymbol{A}^{-1}$ has eigenvalue $1/\lambda$.
   - B. $\boldsymbol{A}^{2}$ has eigenvalue 2λ.

C. If $\boldsymbol{B}$ has eigenvalue λ, then $\boldsymbol{A}$ is similar $\boldsymbol{B}$.
D. $\boldsymbol{A}$ has an eigenvector basis to span an eigenspace of $\boldsymbol{A}$.
E. $\boldsymbol{A} = \boldsymbol{PDP}^{-1}$, where matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ are unique.

13. Let $\boldsymbol{W}$ be a subspace of $\boldsymbol{R}^n$ with an orthogonal basis $\{w_1, .., w_p\}$ and let $\{v_1, .., v_q\}$ be an orthogonal basis for $\boldsymbol{W}^{\perp}$.
A. $\{w_1, .., w_p, v_1, .., v_q\}$ is an orthogonal set.
B. $\text{Span}\{w_1, .., w_p, v_1, .., v_q\} = \boldsymbol{R}^n$.
C. $\dim \boldsymbol{W} + \dim \boldsymbol{W}^{\perp} = n$.
D. $\boldsymbol{x} \in \boldsymbol{W} \cup \boldsymbol{W}^{\perp}$ for any $\boldsymbol{x}$ in $\boldsymbol{R}^n$.
E. $\boldsymbol{W} \cap \boldsymbol{W}^{\perp} = \phi$ (empty set).

14. If $\boldsymbol{A}_{nxn}$ can be spectral decomposed, $\boldsymbol{A} = \lambda_1 \boldsymbol{u}_1\boldsymbol{u}_1^T + \lambda_2 \boldsymbol{u}_2\boldsymbol{u}_2^T + .. + \lambda_m \boldsymbol{u}_m\boldsymbol{u}_m^T$, where λ_i and $\boldsymbol{u}_i$ are eigenvalues and eigenvectors of $\boldsymbol{A}$.
A. $\boldsymbol{A}$ can be any square matrix.
B. $m \le n$.
C. If λ_j is the least eigenvalue, then $\lambda_j \ge 0$.
D. All λ_i are different.
E. All $\boldsymbol{u}_i$ are orthonormal.

15. Find a singular value decomposition $\boldsymbol{A} = \boldsymbol{U}^{\boldsymbol{\Sigma}} \boldsymbol{V}^T$ with $\boldsymbol{U}$ and $\boldsymbol{V}$ being both orthogonal matrices, where $\boldsymbol{A} = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$. Which values are **not** in $\boldsymbol{U}$ or $\boldsymbol{V}$ matrices ?
A. $1/\sqrt{3}$.
B. $1/\sqrt{10}$.
C. $2/\sqrt{10}$.
D. -2/3.
E. 1/3.

16. The following is the pseudo-code for Gauss-Jordan method.

```
for k=1 to n
{
    for j=k+1 to n+1
        akj = akj/X

    for i=1 to n; i is not equal to k
        for j=k+1 to n+1
            aij=aij-(aik)(akj)
}
```

Which of the following statements are correct?
A. the solution is stored in a(i, n), i=1 to n
B. the solution is stored in a(n+1, i), i=1 to n

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C. **X** is a_{nn}

D. **X** is a_{kk}

E. the solution is stored in a(i, n+1), i=1 to n

17. Which of the following transformations are not linear?

A. S: the map in R^3 which rotates points about the x1-axis by an angle $\pi/2$.

B. $T[x_1,x_2,x_3]^T = [x_1+1, x_2-1, x_3]^T$

C. $T[x_1,x_2]^T = [x_1-x_2, x_1+x_2]^T$

D. $T(ax^2+bx+c)=(a+b)x+(b+c)$

E. $T[x]=e^x$

18. Select the value(s) of k so that the matrix

$$\begin{bmatrix} k & 1 \\ k & k \end{bmatrix}$$

is not invertible.

A. 1

B. -1

C. 0

D. no solution

E. k can be any value.

19. Given the following system of linear equations:

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 0 \\ -x_1 - 2x_2 + 3x_3 + 5x_4 &= 0 \\ -x_1 - 2x_2 - x_3 - 7x_4 &= 0 \end{aligned}$$

Which vectors form a basis for the solutions to the system?

A. $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} -1 \\ -2 \\ -1 \\ -7 \end{bmatrix}$ C. $\begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} -1 \\ -2 \\ 3 \\ 5 \end{bmatrix}$ E. $\begin{bmatrix} -4 \\ 0 \\ -3 \\ 1 \end{bmatrix}$

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20. The following is the pseudo code for LU Decomposition.

for i=1, 2, 3, …, $\boldsymbol{a}$

$$L_{i1} = A_{i1}$$

for j=$\boldsymbol{b}$, …, n

$$U_{1j} = \frac{A_{1j}}{L_{11}}$$

for j=2, 3, …, n-1

{

for i=j, j+1, …, n

$$L_{i1} = A_{ij} - \sum_{k=1}^{\boldsymbol{c}} L_{ik}U_{kj}$$

for k=j, j+1, …, n

$$U_{jk} = (A_{jk} - \sum_{i=1}^{\boldsymbol{d}} L_{ji}U_{ik})/L_{jj}$$

}

$$L_{nn} = A_{nn} - \sum_{k=1}^{n-1} L_{nk}\boldsymbol{X}$$

Which of the follow statements are correct?

A. $\boldsymbol{a}$=n-1

B. $\boldsymbol{b}$=1

C. $\boldsymbol{c}$=j-1

D. $\boldsymbol{d}$=j-1

E. $\boldsymbol{X}$=U_{kn}

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