國立中央大學107學年度碩士班考試入學試題

所別： 數學系 碩士班 應用數學組(一般生)
數學系 碩士班 應用數學組(在職生)

科目： 微積分

本科考試禁用計算器

＊請在答案卷(卡)內作答

一共十題。每道十分。均為計算題。請給出計算細節。

Problem 1. (10%) Find $\int_0^{\frac{\pi}{2}} \sin(2x)\sin(3x)dx$.

Problem 2. (10%) Find $\int \sin(\sqrt{x})dx$.

Problem 3. (10%) Find $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$.

Problem 4. (10%) Let

$$f(x) = \int_0^{x^2} t - [t]dt.$$

Find $f'(\frac{1}{2})$. Here $[t]$ denotes the greatest integer less than or equal to t.

Problem 5. (10%) Find the Maclaurin series of $f(x) = \ln(2+x)$.

Problem 6. (10%) Find the volume enclosed between $2x = y^2 + z^2$ and $x^2 + y^2 + z^2 = 3$.

Problem 7. (10%) Find the limit

$$\lim_{x\to 0}\left(\frac{1}{\sin^2(x)} - \frac{1}{x^2}\right).$$

Problem 8. (10%) Find all tangent planes of $x^2 + y^2 + z^2 = x$ which are perpendicular to $x - y - z = 2$ and $x - y + z = 2$.

Problem 9. (10%) Find the maximum value of $f(x, y) = \sin(x) + \sin(y) + \sin(x + y)$.

Problem 10. (10%) Let $n \geq 0$ and $I_n = \int_0^1 \frac{x^n}{1+x}dx$.

(a) Find a formula relating I_n and I_{n+1};

(b) Use the formula you find in (a) to evaluate the following limit:

$$\lim_{n\to\infty}\left(1 - \frac{1}{2} + \frac{1}{3} - \cdots + (-1)^{n-1}\frac{1}{n}\right) = ?$$

參考用