科目 微积分

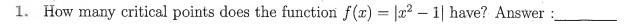
類組別 A3 A4 A7

共/頁第/頁

*請在答案卷內作答

甲、填充題:共 8 題,每題 8 分,共 64 分。請在答案卷上列出題號依序作答。

請注意:本(甲、)部分,共8題,命題型態為填充題,不必詳列計算過程,倘若答案被 包含在演算過程,將被視為試算流程,無法計分。



2. If
$$\lim_{x\to 0} \frac{\sqrt{ax+b}-2}{x} = 1$$
, then $a-b = ?$ Answer:

- 3. Suppose f'' is continuous on [0,1], f(1)=2, f'(1)=2 and the average value of f on [0,1] is 2. Evaluate $\int_0^1 x^2 f''(x) dx$. Answer:
- 4. Find $g'(-\frac{1}{2})$, where g(x) is the inverse of $f(x) = \frac{x^3}{x^2 + 1}$. Answer:
- 5. Find the directional derivative of $f(x,y) = \tan^{-1}(\frac{x}{y})$ at $P_0(1,1)$ in the direction of $v = \frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$. Answer :_____
- 6. Evaluate $\int_0^6 \int_{-1/2}^2 x \sqrt{y^3 + 1} \, dy dx$. Answer:
- 7. Let R be the solid inside $x^2 + y^2 + z^2 = 1$, outside $z = \sqrt{x^2 + y^2}$ and above the xy-plane. Find the volume of R. Answer:
- Evaluate the line integral $\int_C 2xy \, dx + (x^2 + 2x) \, dy$, where C: boundary of the region lying between the graphs of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the circle $x^2 + y^2 = 1$. Answer:

乙、計算、證明題:共3題,每題12分,共36分。須詳細寫出計算及證明過程,否則 不予計分。

- Find any extrema of the function $f(x,y) = e^{-xy/4}$ subject to the constraint $x^2 + y^2 \le 1$.
- 2. For the function

$$f(x,y) = \begin{cases} \frac{-5xy}{x^2 + y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

show that $f_x(0,0)$ and $f_y(0,0)$ both exist, but that f is not differentiable at (0,0).

Determine whether the series $\sum_{n=2}^{\infty} \frac{\ln n}{\ln \ln n}$ converges absolutely or conditionally, or diverges.