科目 微積分 類組別 A2 共 1 頁 第 1 頁

＊請在答案卷內作答

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷上列出題號依序作答。

請注意：本（甲、）部分，共 8 題，命題型態為填充題，不必詳列計算過程，倘若答案被包含在演算過程，將被視為試算流程，無法計分。

1. Find $\lim_{h\to 0}\frac{1}{h}\int_1^{1+h}\sqrt{1+t^2}\,dt$. Answer :_____

2. How many critical points does the function $f(x)=|x^2-1|$ have? Answer :_____

3. Find the number a and b such that $\lim_{x\to 0}\frac{\sqrt{ax+b}-2}{x}=1$. Answer :_____

4. Market has two commodities A and B. The demand equations that relate the quantities demanded x and y to the unit prices p and q of the commodities A and B respectively are given by $x=f(p,q)=\frac{q}{p+q}$, $y=g(p,q)=e^{-(2q+p^2)}$. Are A and B substitute, complementary or neither? Answer :_____

5. Find the slope of the tangent line to the graph of the function $f(x)=(\tan x)^2+e^x$ at $(0,1)$. Answer :_____

6. Find the volume of solid bounded above by by $z=f(x,y)=e^{-x^2}$ and below by the plane region R which is bounded by $y=x$, $x=1$ and $y=0$. Answer :_____

7. The Cobb-Douglas production function for a software manufacturer is given by $f(x,y)=100x^{3/4}y^{1/4}$ where x represents the units of labor (at \$150 per unit) and y represents the units of capital (at \$250 per unit). The total cost of labor and capital is limited to \$50,000. Find x and y that will yield the maximum production level for this manufacturer.
Answer :_____

8. Suppose f'' is continuous on $[0,1]$, $f(1)=2$, $f'(1)=2$ and the average value of f on $[0,1]$ is 2. Evaluate $\int_0^1 x^2 f''(x)\,dx$. Answer :_____

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1 Find any extrema of the function $f(x,y)=e^{-xy/4}$ subject to the constraint $x^2+y^2\le 1$.

2. Evaluate $\int_0^6\int_{x/3}^2 x\sqrt{y^3+1}\,dydx$

3. Determine whether the series $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ converges absolutely or conditionally, or diverges.