類組：電機類　科目：工程數學D(3006)　

※請在答案卷內作答

(各計算題應詳列計算過程，無計算過程者不予計分)

一.　(15分, 計算題)

(一)(7分) Consider a second-order system described by the following differential equation $x'' + 2\zeta\omega_n x' + \omega_n{}^2 x = f(t)$, $\omega_n > 0$, $0 \le \zeta < 1$. We assume that the system is initially passive : $x(0) = x'(0) = 0$. Find the transfer function $\mathrm{H(s) = X(s)/F(s)}$ and the unit impulse response $h(t) = \mathcal{L}^{-1}\{H(s)\}$ if we define $f(t)$ and $x(t)$ are the input and output of the system, respectively.

(二)(8分) Solve $y' + y = g(t)$, $y(0) = 5$, where $g(t) = \begin{cases} 0 \ , & 0 \le t < \pi \\ 3\cos t \ , & \pi \le t \end{cases}$

二.(15分, 計算題)

Let $e^{At} = \begin{bmatrix} (t+1)e^{-t} & (t+1)e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -te^{-t} & -te^{-t} + e^{-2t} & e^{-2t} - e^{-t} \\ te^{-t} & te^{-t} & e^{-t} \end{bmatrix}$

(一)(6分) Find A.

(二)(4分) Solve the initial-value problem $\vec{x}'(t) = A\vec{x}(t)$, $\vec{x}(0) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$.

(三)(5分) Find a particular solution $\vec{x}_p(t)$ of the linear system

$$\vec{x}'(t) = A\vec{x}(t) + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

注意：背面有試題

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三.(10分, 計算題)

Find a formal Fourier series solution of the endpoint value problem

$$\frac{1}{16}\ddot{x} + 4x(t) = f(t), x'(0) = 0, x'(1) = 0 \; and \; f(t) = \pi t, 0 < t < 1$$

四.(10分,計算題)

(一)(5分) Find a solution $y_1(x) = x^r \sum_{n=0}^{\infty} c_n x^n$ $(c_0 = 1, x > 0)$ of the equation

$$x^3 y'' - xy' + y = 0$$

(二)(5分) Use the reduction of order method to find the second solution $y_2(x)$.

五.(8分, 計算題) Find the best least square fit by a linear function to the data {(-1,0), (0,1), (2,3), (3,9)}.

六.(14分, 計算題) Let $A = U\Sigma V^T$, where $U = [\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4] \in \mathbb{R}^{4\times 4}$ and $V = [\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3] \in \mathbb{R}^{3\times 3}$ are orthogonal matrices, and

$$\Sigma = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

(一)(8分) Please find $N(A)$, $R(A)$, $N(A^T)$, and $R(A^T)$.

(二)(6分) Find all the eigenvalues of $A^T A$ and their corresponding eigenspaces.

注意:背面有試題

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In the followings, we denote $\mathbb{R}^{n\times n}$ the set of $n \times n$ matrices with real coefficients, A^T the transpose of a matrix A, $N(A)$ the null space of A, and $R(A)$ the range space of A.

七. (每子題 4 分) For each statement that follows, please answer true or false. In the case of a true statement, you **MUST** explain or prove your answer. In the case of a false statement, a counterexample **SHOULD** be provided.

(一) Let $A \in \mathbb{R}^{m\times n}$. Then $\text{rank}(A) = \text{rank}(A^TA)$.

(二) Let $A \in \mathbb{R}^{n\times n}$ and $A^m = 0$ for some positive integer m. Then all the eigenvalues of A are zero.

(三) Let A be a nonsingular matrix and $A^{-1} = A$. Then A is either the identity matrix I or $-I$.

(四) Any two square matrices with the same trace and determinant are similar.

(五) Any two similar matrices have the same determinant and trace.

(六) Let $A \in \mathbb{R}^{m\times n}$ and $N(A) = \{0\}$. Then the column vectors of A are linearly independent.

(七) Let $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times m}$. Then $AB = 0$ implies that $BA = 0$.