國立中央大學109學年度碩士班考試入學試題

所別：光電類　　　　　　　　　　　　　　　　　　共3頁　第1頁

科目：工程數學

本科考試可使用計算器，廠牌、功能不拘　　　　　　＊請在答案卷(卡)內作答

一、選擇題 (50 %)(選擇題答案請作答於答案卷內，每小題各佔5%)

(1) For an equation given as $\frac{dy}{dx}=\frac{2+\cos(x)}{(y-1)^2}$ with $y(0)=3$, what can be $y(x)$?

(A) $y(x)=1+[2x+\cos(x)+8]^{1/3}$
(B) $y(x)=1+[2x+\sin(x)+8]^{1/3}$
(C) $y(x)=1+[2x+\sin(x)+8]^{1/2}$
(D) $y(x)=1+[\ x+\sin(x)+8]^{1/3}$
(E) None of the above

(2) If $\frac{df(t)}{dt}=4f(t)$, with $f(0)=-5$, what is f(t)?

(A) $-4e^{4t}$　(B) $-2e^{4t}$　(C) $-5e^{4t}$　(D) $-5e^{-4t}$　(E) $5e^{4t}$

(3) A radioactive material with the total amount, $M(t)$, is decaying with the rate: $dM(t)/dt=-k\cdot M(t)$, where t is time and k is some positive constant with the unit of **year^{-1}**. If the "half-life" is defined as the amount of time (τ) necessary for one-half of the initial amount of material to disappear, meaning $M(\tau)=M(0)/2$. If the radioactive material is of the $k=4.95\times10^{-11}$, what is 2τ for the material?

(A) 2.8×10^{10} years　(B) 1.4×10^{10} years　(C) 1.4×10^{9} years
(D) 4.95×10^{10} years　(E) 4.95×10^{11} years

(4) If $x^2\frac{dy}{dx}=y^2-xy+x^2$ and $y=2$ when $x=1$, what is y when $x=1000$?

(A)100　(B)500　(C)0　(D) 1　(E)10

(5) For the differential equation: $(2x+2y^2)+(4xy+3y^2)\frac{dy}{dx}=0$ and c is an arbitrary constant, what is the relationship between x and y?

(A) $2x^2+2xy^2+y^3=c$　(B) $x+2xy^2+y^3=c$　(C) $x^2+xy^2+y^3=c$
(D) $2x+2xy^2+y^3=c$　(E) $x^2+2xy^2+y^3=c$

參考用

(6) If f is a function of t and $\frac{df(t)}{dt}=\frac{t}{f(t)}+\frac{f(t)}{t}$ with $f(2)=1$, what is $f(200)$?

(A) 1　(B) $\sqrt{17}$　(C) 0　(D) $100\sqrt{17}$　(E) 100

(7) For the differential equation: $(2x+1+2y^2)+(4xy+3y^2)\frac{dy}{dx}=0$, if $y=-1$ when $x=0$, what can be the value of x when $y=1$?

(A) -3　(B) 2　(C) 1　(D) 0　(E) -2

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(8) In a semiconductor p-n junction under forward bias, the distribution of hole concentration, $p(x)$, in the n-region is determined by the following differential equation: $\frac{d^2p(x)}{dx^2} = \frac{p(x)}{L^2}$, where L is referred as the diffusion constant. If $p(\infty) = 0$ and $p(0) = C$, what can be the form of $p(x)$?

(A) $p(x) = C\,e^{-x/L}$ (B) $p(x) = C\,e^{x/L}$ (C) $p(x) = C\,e^{-ix/L}$

(D) $p(x) = C\,e^{-x/L} + C\,e^{x/L}$ (E) $p(x) = C\,e^{-ix/L} + C\,e^{ix/L}$

(9) In quantum mechanics, the wavefunction of a particle, $\psi(x)$, in free space is governed by the Schrodinger equation, given as $\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$, where $\hbar$ is the Planck constant; m, E, x is the mass, the energy, the position of the particle, respectively. What is the form of $\psi(x)$?

(A) $e^{\pm\frac{\sqrt{2mE}}{\hbar}x}$ (B) $e^{\pm i\frac{\sqrt{2mE}}{\hbar}x}$ (C) $e^{\pm i\frac{\sqrt{2mE}}{2\hbar}x}$ (D) $e^{\pm i\frac{\sqrt{mE}}{\hbar}x}$ (E) $e^{\pm i\frac{\sqrt{mE}}{2\hbar}x}$

(10) Repeating question (9), if the particle is placed within an infinite well shown blow, the Schrodinger equation is modified as follows: $\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$, where $V(x) = 0$ at $\frac{-a}{2} < x < \frac{a}{2}$; $V(x) = \infty$ at $x \le \frac{-a}{2}$ and $x \ge \frac{a}{2}$. Using the boundary conditions: $\psi\left(\frac{-a}{2}\right) = \psi\left(\frac{a}{2}\right) = 0$ and assuming C is a certain constant and $k = \frac{\sqrt{2mE}}{\hbar}$, what can be the combination of $\psi(x)$ and E?

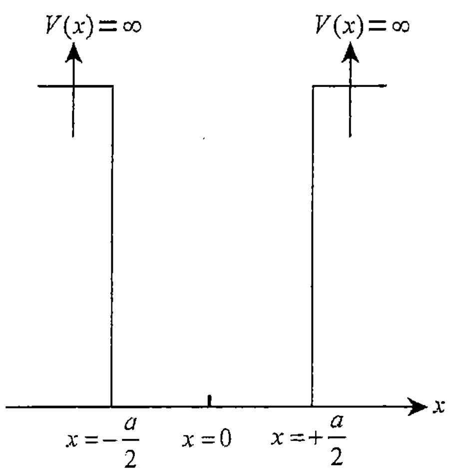
(A) $\psi(x) = C \cdot cos(kx)$ with $E = \frac{2\pi^2\hbar^2}{ma^2}$;

(B) $\psi(x) = C \cdot sin(2kx)$ with $E = \frac{\pi^2\hbar^2}{ma^2}$;

(C) $\psi(x) = C \cdot sin(kx)$ with $E = \frac{2\pi^2\hbar^2}{ma^2}$;

(D) $\psi(x) = C \cdot cos(kx)$ with $E = \frac{2\pi\hbar^2}{ma^2}$;

(E) $\psi(x) = C \cdot cos(2kx)$ with $E = \frac{\pi^2\hbar^2}{ma^2}$.



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二、計算題 (50 %) (詳細計算過程與答案，請作答於答案卷內)

10% (1) The Legendre polynomials $\boldsymbol{P}_n(x)$ are orthogonal with respect to the weight function $w(x)=1$ on the ontervl (-1,1) and

$$\|P_n\|^2 = \int_{-1}^{1}[P_n(x)]^2 dx = \frac{2}{2n+1}, \quad \text{where } n\geq 0.$$

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \frac{1}{2}(3x^2 - 1), \dots .$$

Therefore, the coefficients in the Fourier-Legendre series, $\sum_{n=0}^{\infty} C_n P_n(x)$, for an arbitrary function $f(x)$ are given by $C_n = \frac{2n+1}{2}\int_{-1}^{1} f(x)P_n(x)\,dx, \quad n \geq 0.$

Now, it is given that

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x, & -1 \leq x \leq 0. \end{cases}$$

Find the Fourier-Legendre series for $f(x)$.

10% (2) In the Cartesian coordinates, the differential Laplacian operator ∇^2 can be expressed as

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Now, express the differential Laplacian operator ∇^2 in the (u, v, z) parabolic cylindrical coordinating system whose transformation from the Cartesian (x, y, z) is

$$\begin{cases} xy &= u \\ x^2 - y^2 &= v. \\ z &= z \end{cases}$$

(3) Consider a finite wave train defined by:

$$f(t) = \begin{cases} \cos \omega_0 t & |t| < \dfrac{N\pi}{\omega_0} \\ 0 & |t| > \dfrac{N\pi}{\omega_0} \end{cases}$$

10% (a) Perform appropriate Fourier transform of the finite wave train and plot the function out. No to specify the value of the maximum amplitude and the location where it occurs.

5% (b) Evaluate the quality factor Q ($\equiv \frac{\Delta\omega}{\omega_0} = \frac{\omega_0 - \omega}{\omega_0}$) and describe how it varies with N.

15% (4) Diagonalize the matrix $\mathbf{A}$ such that the diagonalized matrix $\mathbf{D} = \mathbf{X}^{-1}\mathbf{A}\mathbf{X}$ is diagonal.

Then, find $\mathbf{A}^{10}$.

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & -2 \\ 2 & 4 & 1 \\ 2 & 1 & 4 \end{pmatrix}$$