

國立中央大學 109 學年度碩士班考試入學試題

所別： 統計研究所 碩士班 不分組(一般生)

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統計研究所 碩士班 不分組(在職生)

科目： 基礎數學

*計算題需計算過程，無計算過程者不予計分

本科考試可使用計算器，廠牌、功能不拘

*請在答案卷(卡)內作答

1. Calculate

(a) $\int_{-\infty}^{\infty} x^2 e^{-(x-2)^2/6} dx$, (b) $\int_0^1 x^9 (1-x)^7 dx$. (9+9=18%)

2. Test for convergence:

(a) $\sum_{n=1}^{\infty} \frac{\ln n}{2n^3 - 1}$, (b) $\sum_{n=1}^{\infty} \frac{5n^2 - 2n + 1}{n^3 + 2n}$. (8+8=16%)

3. Find the minimum and maximum value of $x^2 + y^2 + z^2$ subject to

the constraint conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $z = x + y$. (16%)

4. Let

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 17 \end{bmatrix} \text{ and define } T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{bmatrix},$$

then $A = T^T T$.

If λ_i , $i=1,2,3$ are the eigenvalues of matrix A, find $\sum_{i=1}^3 \lambda_i$ and $\prod_{i=1}^3 \lambda_i$. (8+8=16%)

5. (a) Verify, when A, D are symmetric matrices such that the inverses (10%)

which occur in the expressions exist, that

$$\begin{pmatrix} A & B \\ B' & D \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} + FE^{-1}F' & -FE^{-1} \\ -FE^{-1} & E^{-1} \end{pmatrix}$$

where $E = D - B' A^{-1} B$, $F = A^{-1} B$

(b) Find the inverse of the following matrix (8%)

$$\begin{pmatrix} 2 & 0 & 0 & 3 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 1 & 2 \\ 3 & 5 & 2 & 6 \end{pmatrix}$$

6. Suppose that $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

(a) Derive the orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix. (8%)

(b) Derive A^{10} . (8%)

參考用

