

國立中央大學 110 學年度碩士班考試入學試題

所別： 天文研究所碩士班 不分組(一般生)

共 2 頁 第 1 頁

科目： 應用數學

本科考試禁用計算器

*請在答案卷(卡)內作答

※計算題需計算過程，無計算過程者不予計分

- Variables x are uniformly distributed between interval a and b .
 - What is the corresponding probability density function $p(x)$? [3%]
 - What is the normalization constant A such that $\int Ap(x)dx = 1$? [3%]
 - What is the corresponding mean for this distribution? [3%]
 - What is the corresponding median for this distribution? [3%]
 - What is the corresponding variance for this distribution? [4%]
 - What is the corresponding skewness for this distribution? [4%]

- Consider a linear regression in the form of $y = ax + b$ with data points of (x_i, y_i) , where $i = 1, \dots, N$.
 - Derive the analytical expression of slope a and intercept b . [14%]
 - Calculate a and b for the following data points: $(-2, -1)$, $(1, 1)$ and $(3, 2)$ [7%]
(NOTE: you can express your answers as ratio instead of decimal number, for example 11/10 instead of 1.1)
 - The data given in (b) can also be fitted with $y = ax$. Which regression, $y = ax + b$ or $y = ax$, can fit the data better? Explain or argue your answer. [7%]

- A measurement Y depends on variables (a, b, c) , the standard deviation of these variables are known: $(\sigma_a, \sigma_b, \sigma_c)$. Assume that the covariance terms can be ignored, answer the following:
 - What is the general equation for σ_Y based on propagation of error? [2%]
 - If $Y = Zab - c$, where Z is a constant, what is the expression of the error term for Y ? [3%]
 - If $Y = a/b + e^c$, where e is natural logarithm. What is the expression of the error term for Y ? [3%]

Now assume that the covariance term for a and c , σ_{ac} , cannot be ignored.

 - What is the expression of the error term for Y in (b)? [4%]

注意:背面有試題

國立中央大學 110 學年度碩士班考試入學試題

所別： 天文研究所碩士班 不分組(一般生)

共 2 頁 第 2 頁

科目： 應用數學

本科考試禁用計算器

*請在答案卷(卡)內作答

4. Write down the Taylor series expansion, up to first-order approximation, for the following function at $x = 0$.

(a) $f(x) = e^x - 1$. [5%]

(b) $f(x) = \ln(1 + x)$. [5%]

(c) $f(x) = (1 + x)^n$. [5%]

5. Consider the following matrix:

$$\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$$

Calculate or find out the following:

(a) The determinant of \mathbf{A} , $\det(\mathbf{A})$. [2%]

(b) The transpose of \mathbf{A} , \mathbf{A}^T . [3%]

(c) The inverse of \mathbf{A} , \mathbf{A}^{-1} . [3%]

(d) Symmetric matrix \mathbf{S} and skew-symmetric matrix \mathbf{K} such that $\mathbf{A} = \mathbf{S} + \mathbf{K}$. [6%]

(e) Two eigenvalues of \mathbf{A} . [5%]

(f) The corresponding two eigenvectors. [6%]

注意:背面有試題