國立中央大學110學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 製造與材料組(一般生)　　共2頁 第1頁
機械工程學系光機電工程 碩士班 光機組(一般生)
能源工程研究所 碩士班 不分組(一般生)

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　*請在答案卷(卡)內作答

※計算題需計算過程，無計算過程者不予計分

1. Find the solutions for ordinary differential equations. (ODE)

(a) (5%) Find the solution for $y''-3y'-4y=0,\ y(0)=2,\ y'(0)=1$

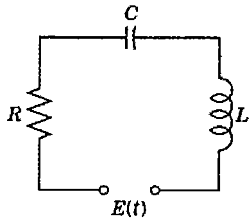
(b) (5%) Find the solution for $x^2y''+2xy'-6y=0,\ y(1)=0.5,\ y'(1)=1.5$

(c) (5%) Find a basis of solutions by the Frobenius method of the following ODE:

$$(x+1)^2y''+(x+1)y'-y=0.$$

2. Modeling an *RLC*-circuit and obtain steady-state current.
Kirchhoff's Voltage Law says that the voltage drop in a closed-loop circuit is zero.

(a) (5%) Based on this law, model the current, $i(t)$, for the circuit shown in the following figure.



(b) (5%) Obtain the "steady-state" current in the *RLC*-circuit when R=50 Ω (Ohm), L = 30 H (Henry), C = 0.025 F (Farad) , and E = 200 sin(4t) V (Volt)

Hint: The voltage drop for a current $i(t)$ across a resistor of resistance R is $Ri(t)$, across an inductor of inductance L is $L\frac{di}{dt}$, and across a capacitor of capacitance C is Q/C, where Q is the charge and the relation between $Q(t)$ and $i(t)$ is $Q(t)=\int i(t)dt$.

3. (5%) Determine the existence and uniqueness of the solutions to the system

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 \\ 3 & -7 & 8 & -5 & 8 \\ 3 & -9 & 12 & -9 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}=\begin{bmatrix} -5 \\ 9 \\ 15 \end{bmatrix}$$

4. (5%) The traffic flow problem can be described by the following table. Please determine the general flow pattern for the network

Intersection	Flow in	=	Flow out
A	300+500	=	x_1+x_2
B	x_2+x_4	=	300+ x_3
C	100+400	=	x_4+x_5
D	x_1+x_5	=	600

注意:背面有試題

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5. (a) (3%) Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, Are u and v eigenvectors of A?,
(b) (4%) Show that 7 is an eigenvalue of matrix A in **(a)**, and find the corresponding eigenvectors.
(c) (8%) Find a formula for A^k, $k \geq 1$ (Hint: given that $A = PDP^{-1}$, P and D matrix can be obtained from eigenvectors and eigenvalues of matrix A)

6. (10%) Let $f(t)$ be a periodic function, $f(t) = f(t+p)$ with period p. Denote $L[f(t)]$ as the Laplace transform of $f(t)$. Prove $L[f(t)] = \frac{\int_0^p e^{-st} f(t)dt}{1-e^{-sp}}$.

7. Definition: The Fourier series expansion of a function $f(t)$ is given by

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)], \omega_0 = \frac{2\pi}{T}. \quad (1)$$

Function $f(t)$ is given by $f(t) = \begin{cases} 0, 0 \leq t < \pi \\ 2, \pi \leq t < 2\pi \end{cases}$ and $f(t) = f(t+2\pi)$. Expand $f(t)$ by Fourier series.
(a) (3%) Find the (fundamental) period T of $f(t)$.
(b) (4%) Find the values of $a_0,\ a_1,\ a_2, a_3,\ b_1,\ b_2,\ b_3$.
(c) (3%) Will the Fourier series converge to $f(t)$? Explain your reasons within 30 words.
(d) (5%) Can one obtain identical Fourier series of any function $g(t)$ by using $2T$ and T in Equation (1)? Explain your reasons within 30 words.

8. It is given grad $f = \nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$, div$\vec{v} = \nabla \vec{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$, and curl$\vec{v} = \nabla \times \vec{v} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix}$, where f and v_1, v_2 and v_3 are the scalar functions of x, y and z.

(i) $\underline{\nabla \times (\nabla f) = 0}$ (2)

states "Gradient fields are irrotational. That is, if a continuously differentiable vector function is the gradient of a scalar function f, then its curl is a zero vector."

(ii) $\underline{\nabla(\nabla \times \vec{v}) = 0}$ (3)

states "the divergence of the curl of a twice continuously differentiable vector function v is zero."

(a) **(6%), (6%)** Please first **prove** Equations (2) and (3), respectively

(b) **(6%), (7%)** Please give their (Equations (2) and (3)) **matching engineering (or physical) examples** with **discussion**, respectively.

注意:背面有試題