

國立中央大學 111 學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 系統組(一般生)

共 2 頁 第 1 頁

科目： 工程數學(含程式設計)

1. (10%) The Legendre polynomials, $P_m(x)$, $m = 0, 1, 2, \dots$ are given by $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$, $P_3(x) = \frac{1}{2}(5x^3 - 3x)$, and so forth. It is known that $P_m(x)$ is orthogonal to $P_n(x)$ for $m \neq n$ on interval $-1 \leq x \leq 1$. Expand $f(x) = 3x^2 - 2x + 2$ by Legendre polynomials, i.e., $\sum_{n=0}^N a_n P_n(x)$.

- (a) (i) (3%) Find a_0, a_1, a_2 , and a_3 as $N \rightarrow \infty$.
 (ii) (2%) Is the expansion complete?
 (b) (i) (3%) Find a_0, a_1, a_2 , and a_3 as $N = 3$.
 (ii) (2%) Is the expansion complete?

2. (15%) A function $f(t)$ is expanded using Fourier series.

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(\omega_n t) + b_n \sin(\omega_n t)], \omega_n = \frac{2\pi n}{T}, T = 2\pi \quad (1)$$

, where

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt, \quad a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(\omega_n t) dt,$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(\omega_n t) dt; n = 1, 2, \dots$$

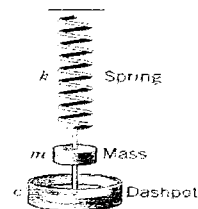
Suppose the coefficients are given by $a_0 = 1, a_n = 0$ for $n > 0$, and $b_n = 2^{-n}, n > 0$.

- (a) (3%) What are the (fundamental) period and fundamental frequency, respectively?
 (b) (3%) $f(t)$ is an even function, an odd function, or neither?
 (c) (3%) Find the average value of $f(t)$ over $t = [0, 2\pi)$.
 (d) (3%) Does $f\left(t + \frac{3T}{2}\right)$ equal to $f(t)$? Briefly give your reasons within 20 words.
 (e) Eq. (1) can be expressed as $f(t) = \sum_{n=0}^{\infty} [c_n \cos(n\omega_0 t + \phi_n)], 0 \leq \phi_n < 2\pi$.
 Find c_0 and c_1 . (3%)

3. (15%) Solutions for ordinary differential equations (ODEs)

- (a) (5%) Find the solution for the ODE: $y'' - k^2 y = 0$ ($k \neq 0$), $y(0) = 1, y'(0) = 1$
 (b) (5%) Find the solution for the ODE: $y'' + 4y' + 5y = e^{-t} \cos(t), y(0) = 0, y'(0) = 1$
 (c) (5%) Find a basis of solutions by the Frobenius method of the following ODE:
 $(x + 1)^2 y'' + (x + 1) y' - y = 0$.

4. (10%) For the mass-spring system, as shown in the following figure, find its motion as a function of time, $y(t) = ?$ If the mass, m , is 0.25 kg, damping, c , is zero, spring constant, k , is 2.25 kg/sec², and driving force is $\cos(t) - 2\sin(t)$. Assuming zero initial displacement and velocity, $y(0) = 0, y'(0) = 0$. For what frequency of the driving force would you get resonance?



注意:背面有試題

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5. (5 %) Suppose that in a weight-watching program, Sam, weighing 180 lb, burns 350 cal/hr in walking (3 mph), 500 cal/hr in bicycling (13 mph), and 950 cal/hr in jogging (5.5 mph). He plans to exercise 4 days a week following to Mon (1.0, 0, 0.5), Wed (1.0, 1.0, 0.5), Fri (1.5, 0, 0.5), and Sat (2.0, 1.5, 1.0), where 3 numbers in each bracket give the time (hrs) taken to walk, bicycle, and jog, respectively, on that day. Please give the **(4 by 1) column vector** that shows the calories burned on Mon, Wed, Fri, and Sat in a week.

6. (20%)

(a) (5%) Find the inverse of the matrix, $\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{8} & 0 & 0 \end{bmatrix}$, by **Gauss-Jordan elimination**.

(b) (8%) Please explain or address why Gauss-Jordan elimination can be used to calculate the inverse of a matrix.

(c) Let $A = [a_{jk}]$ be a nonzero square matrix of dimension $n \times n$. The problem of finding nonzero \vec{x} 's and λ 's that satisfy the vector equation

$$A\vec{x} = \lambda\vec{x} \quad (2)$$

is called an eigenvalue problem.

(i) (4%) Please interpret the meaning of Eq. (2).

(ii) (3%) What do you find from the matrix multiplication $\begin{bmatrix} 6 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \end{bmatrix}$?

7. (10%) 請撰寫一個程式，它會要求使用者『輸入』一個卡氏座標系統(Cartesian coordinate system)中的一點(x, y)，其中 x 和 y 變數以實數宣告。之後該程式會依輸入之值，『顯示』其所對應的點位於哪一個象限上。請注意：座標的位置有 7 種不同的情況：第 I 象限、第 II 象限、第 III 象限、第 VI 象限、在 X 軸上、在 Y 軸以及原點。程式碼限定以 C、C++、Visual Basic 或 Fortran 撰寫，請在答案卷上註明使用的程式語言。

8. (15%) 如果整數的因數(包括 1)加起來的和等於它自己，稱為完全數(Perfect number)。例如：

6 的因數(可完全整除 6)有 1, 2, 3，而 $1 + 2 + 3 = 6$ ，則 6 就是一個完全數。

請撰寫一個程式，尋找並在螢幕上顯示 1~8888 內所有完全數。不需實際算出，只要寫出程式碼，限定以 C、C++、Visual Basic 或 Fortran 撰寫，所有變數均以整數宣告，並在答案卷上註明使用的程式語言。

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