多選題每題 5 分，共 65 分，答錯每個選項倒扣 1 分，扣至該大題（多選題）零分為止

For question 1~4, matrix $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ describes a binary relation **R.**

1. About relation **R**, which of the following statements are true?
   A) **R** is reflexive. B) **R** is anti-symmetric. C) **R** is transitive.
   D) **R** is a partial ordering relation. E) The symmetric closure of **R** is transitive.

2. Consider different closures of **R.** Which of the following statements are true?
   A) 4 more '1's must be added to **A** to make **R**'s reflexive closure.
   B) 6 more '1's must be added to **A** to make **R**'s symmetric closure.
   C) 3 more '1's must be added to **A** to make **R**'s transitive closure.
   D) The reflexive closure of **R** is a partial ordering set.
   E) The symmetric closure of **R** is an equivalence relation.

3. If **G** is a graph representation of **R**, which of the following statements are true?
   A) **G** is weakly connected.
   B) the longest simple path of **G** is length 4.
   C) The reflexive, symmetric, and transitive closure of **R** has 2 connected components.
   D) There is a Hamilton path in the transitive closure of **R**.
   E) $\mathbf{R}^{-1}$ is strongly connected.

4. Let **S** be the symmetric and reflexive closure of **R** and each element is a proposition. If **S** reflects Boolean operators' behavior, what are possible operators?
   A) $\wedge$ B) $\vee$ C) $\neg$ D) $\rightarrow$ E) $\leftrightarrow$

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5. Which are sufficient but not necessary conditions for the corresponding goals?
   A) "Graph G_1 and G_2 are isomorphic" for "G_1 and G_2 both have Euler circuits."
   B) growth order function f and g, "g is $O(f)$ but not $\Theta(f)$" for "g is $o(f)$".
   C) "Existing an equivalence relation **R** on set **S**" for "**S** has a partition based on **R**".
   D) An infinite set of predicates **P**, "Existing a well order on **P**" for "using mathematic induction to prove all predicates in **P**".
   E) "P is false" for "P→Q is true "

6. Consider a set **S** of n nodes interconnecting to form a graph **G**. If two nodes "directly connect" to each other, there is an undirected edge between them. Let D(a) denote the degree of a node a. Which of the following are correct statements?
   A) If **G** is connected, there must be a simple path of length n.
   B) $\sum_{i\in S} D(i)$ is even. C) If **G** is connected, $\sum_{i\in S} D(i) \geq 2n$.
   D) "$\exists a, b, (a \neq b) \wedge (D(a) = D(b))$" is true.
   E) If **G** is connected, the transitive and reflexive closure of "direct connecting" relation can form a complete graph.

7. To analyze the complexity of the following procedure P, We will use the following assumptions: Suppose P and B are both procedures. B take $\theta(\sqrt{m})$ time to compute, where m is the size of B's input; each statement line in and outside the loop counts 1 step.

```
Procedure P( array1[a_1, a_2, ..., a_n])
1. if n<9 exit.
2. call B(array1[a_1, a_2, ..., a_n])
   declare new empty array2, array3, array4;
3. for (i=1 to n)
4. { if ((i mod 9)=0)   insert a_i into array2;
5.   if ((i mod 9)=3)   insert a_i into array3;
6.   if ((i mod 9)=6)   insert a_i into array4 }
8. call P( array2 );
9. call P( array3 );
10. call P (array4);
```

Which of the following relations on F_n can describe the complexity of procedure P with respect to problem size n?

A) $F_n = 3F_{n/3} + \theta(\sqrt{n})$ B) $F_n = 3F_{n/3} + F_n + \theta(n)$ C) $F_n = 9F_{n/9} + \theta(\sqrt{n})$

D) $F_n = 3F_{n/9} + \theta(n)$ E) $F_n = 3F_{n/9} + \theta(n^{1/2})$

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8. What can be the time complexity level of the procedure P in question 7?

   A) $O(n^{\sqrt{2}})$ B) $O(n\log n)$ C) $O(n^{\log n})$ D) $O(n^{1/2})$ E) $O(\sqrt{n}\log n)$

9. An alphabet set $\{\alpha_1,\alpha_2,...,\alpha_8\}$ is used to form a string. A legal string cannot have consecutive $\{\alpha_1\alpha_1,\alpha_1\alpha_2,\alpha_2\alpha_1,\alpha_2\alpha_2\}$ in any part of the string. Suppose P_{n-1} is the number of valid string of length n. Which of the following are true?

   A) $P_n=2P_{n-1}+6^n, n\geq 2$ B) $P_n=6P_{n-1}+12P_{n-2}, n\geq 2$

   C) $P_n=8P_{n-1}-4, n\geq 2$ D) $P_0=8, P_1=48$ E) $P_0=8, P_1=64$

10. When using the generating function $g(z)$ to solve P_n in the previous question, which of the following are true?

   A) $g(z)(1-6z-12z^2)=60z+8$ B) $g(z)(1-6z+12z^2)=12z+8$

   C) $g(z)=\dfrac{4+2\sqrt{21}}{1-(3+\sqrt{21})z}+\dfrac{4-2\sqrt{21}}{1-(3-\sqrt{21})z}$

   D) $g(z)=\dfrac{4+(18/\sqrt{21})}{1-(3+\sqrt{21})z}+\dfrac{4-(18/\sqrt{21})}{1-(3-\sqrt{21})z}$

   E) $P_n=((4+18/\sqrt{21})\times(3+\sqrt{21})^n)+((4-18/\sqrt{21})\times(3-\sqrt{21})^n)$

11. Which of the following statements about symmetric matrices are true?

   A) The symmetric matrix $\boldsymbol{A}$ has a complete set of orthonormal eigenvectors even though it has repeated eigenvalues.

   B) For the symmetric matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{A}+\boldsymbol{B}$ and $\boldsymbol{AB}$ are both symmetric.

   C) If the rank of a symmetric matrix, $\boldsymbol{A}_{n\times n}$, is $r<n$, then its n-r eigenvalues are zero.

   D) The eigenvalues of symmetric matrices are real values. In other words, they cannot be complex values.

   E) None of the above is true.

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12. Which of the following statements about the matrix $A_{m \times n}$, $m<n$ are true?

A) The sum of its nullity and its rank is m.

B) If its rank is equal to m, then AA^T must be invertible.

C) The number of independent columns and the number of independent rows in the matrix A can be different.

D) After applying Gaussian elimination on A, the number of zero rows must be m-r, where r is the rank of A.

E) None of the above is true.

13. Which of the following about eigenvalues/eigenvectors are true?

A) If an 8x8 square matrix has 8 positive pivots after applying Gaussian elimination, then its eigenvalues cannot be negative.

B) For the two square matrices A and B, both of which are diagonalizable, if $AB = BA$, then A and B have the same eigenvalues.

C) If two square matrices A and B are similar, i.e., $A = MBM^{-1}$, then A and B have the same eigenvalues.

D) For a square matrix A, the matrices AA^T and A^TA have the same eigenvalues.

E) None of the above is true.

國立中央大學112學年度碩士班考試入學試題

所別：資工類　　　　　　　　　　　　　　　　　　　　　　

科目：離散數學與線性代數

單選題每題5分，共35分，答錯一題倒扣2分，扣至該大題（單選題）零分為止

14. Given a 4 by 4 matrix, $\boldsymbol{A}$, as below, find its inverse matrix, $\boldsymbol{A}^{-1}$.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 2 & 5 & 1 & -1 \\ 1 & 3 & 2 & 1 \\ 2 & 4 & 1 & -2 \end{bmatrix}$$

The sum of all the elements of $\boldsymbol{A}^{-1}$ is b. Rounding $|b|$ to the nearest integer, c. What is the value of mod(c,5), where mod(.) is the modulo operation. (For example, mod(5,2)=1.)

A) 0　B) 1　C) 2　D) 3　E) 4

15. For the following Matlab code:

```
a=11;                    %Set 'a' as the number of rows/cols. in a square matrix A
A=ones(a,a);             %Set A as an 11x11 all-one matrix (All the elements are 1)
for m=1:a
    A(m,m)=0.9;          %Set all the elements on the main diagonal of A as 0.9
end
d=det(A);                %Calculate the determinant of A
c=abs(d*(10^a)+7);       %Adjust the value, and abs(.) is the absolute-value func.
answer=mod(c,5);         %mod(.) is the modulo operation
```

What is the value of "answer"?

A) 0　B) 1　C) 2　D) 3　E) 4

16. The following matrix $\boldsymbol{A}$ is the transformation matrix of a linear operator A. Find the linear transformation matrix $\boldsymbol{B}$ that represents the linear operator A relative to the basis $[1, 1, 0]^T$, $[0, 1, 1]^T$ and $[1, 2, 2]^T$.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & -4 \\ 1 & -2 & 2 \end{bmatrix}$$

'b' is obtained by summing up all the elements in $\boldsymbol{B}$, taking the absolute value and then rounding to the nearest integer. What is the value of mod(b,5), where mod(.) is the modulo operator?

A) 0　B) 1　C) 2　D) 3　E) 4

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17. Consider the subspace S of $\mathbf{R}^4$ spanned by the vectors:
$\mathbf{v}_1$=[1, 1, 1, 1]^T, $\mathbf{v}_2$=[1, 1, 2, 4]^T, $\mathbf{v}_3$=[1, 2, -4, -3]^T. Apply the Gram-Schmidt method starting from $\mathbf{v}_1$, then $\mathbf{v}_2$ and finally $\mathbf{v}_3$ to obtain the orthogonal basis of S: $\mathbf{u}_1$=[1, 1, 1, 1]^T, $\mathbf{u}_2$=[-1, a, b, c]^T, $\mathbf{u}_3$=[1, d, e, f]^T, where a, b, c, d, e and f are all integers (rounded to the nearest ones if necessary). What is mod(|a+b+c+d+e+f|,5), where mod(.) is the modulo operator?
A)0 B) 1 C) 2 D) 3 E) 4

18. Given that $\boldsymbol{A} = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$, $\boldsymbol{B} = \mathbf{A}^{12}$. 'b' is obtained by summing up all the elements in $\boldsymbol{B}$. What is mod (|b|,5), where mod(.) is the modulo operator?
A)0 B) 1 C) 2 D) 3 E) 4

19. Find the least squares solution of the system $\boldsymbol{Ax} = \boldsymbol{b}$, in which

$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\boldsymbol{b}$=[4, -1, 0, 1]^T.

c is the rounding (or nearest) integer of the sum of the elements in $\boldsymbol{x}$. What is the value of mod(|c|,5), where mod(.) is the modulo operator?
A)0 B) 1 C) 2 D) 3 E) 4

20. Find an orthogonal matrix $\boldsymbol{P}$ that diagonalizes $\boldsymbol{A}$ as shown below. That is,

$\mathbf{P}^T\boldsymbol{AP} = \boldsymbol{D}$, where $\boldsymbol{D}$ is a diagonal matrix.

$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

The product of the three elements in the first row of $\boldsymbol{P}$ is b. The product of the three elements in the main diagonal of $\boldsymbol{D}$ is d. c is the rounding (or nearest) integer of $|b \times d \times 6|$. What is the value of mod(c,5), where mod(.) is the modulo operator?
A)0 B) 1 C) 2 D) 3 E) 4

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