多重選擇題，共20題，每題5分 答錯一個選項倒扣1分，倒扣至本大題(即多選題)0分為止。

1. If a linear transformation from a vector space V to another vector space W is one-to-one, which of the following statements is/are true?
   (A) It is onto;
   (B) dim(V) = dim(W);
   (C) The null space of this transformation contains only the zero vector;
   (D) It is invertible;
   (E) All of the above.

2. Consider a subset $S = \{ (1, 2, 1), (2, -1, 1)\}$ of R^3, which of the following statements is/are true?
   (A) The set S spans a subspace in R^3;
   (B) Such a subspace is the z = 1 plane in R^3;
   (C) The set S is a linearly independent set;
   (D) The two vectors in S are orthogonal in the subspace;
   (E) If we add another vector (0, 0, 1) into S, this set can span R^3.

3. Which of the following processes is/are linear?
   (A) Fourier transform of real-valued functions on R;
   (B) Laplace transform of real-valued functions on R;
   (C) Determinant calculation of real $n\times n$ matrices;
   (D) Inner product of an arbitrary vector $\boldsymbol{x}$ and a fixed vector z in R^n;
   (E) Norm calculation of vectors in R^n.

4. What is the rank of the matrix:

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}?$$

   (A) 0;
   (B) 1;
   (C) 2;
   (D) 3;
   (E) 4.

5. Find the determinant of the symmetric Pascal matrix:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}.$$

   (A) 0;
   (B) 1;
   (C) 2;
   (D) 3;
   (E) 4.

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6. Find the parabola $y = a + bx + cx^2$ that comes closest (least squares error) to the data points: $(x, y) = (-2, 0), (-1, 0), (0, 1), (1, 2)$, and $(2, 0)$.
   (A) $a = 40/35, b = 0, c = 1/7$;
   (B) $a = 41/35, b = 1/5, c = -2/7$;
   (C) $a = 40/35, b = -1/5, c = 2/7$;
   (D) $a = 41/35, b = 0, c = -2/7$;
   (E) $a = 40/35, b = 1/5, c = 1/7$.

7. Which of the following sets consists of the eigenvalues of $A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$?
   (A) $\{3, -4\}$;
   (B) $\{3, 4\}$;
   (C) $\{5, -7\}$;
   (D) $\{4, -3\}$;
   (E) $\{2, -4\}$.

8. Consider two matrices $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 \\ 3 & 0 \\ -3 & 4 \end{bmatrix}$. Which of the following statements about the inner product $\langle A, B \rangle$, and their orthogonality is/are true?
   (A) $\langle A,B \rangle = 0$, orthogonal;
   (B) $\langle A, B \rangle = 6$, not orthogonal;
   (C) $\langle A, B \rangle = 1$, not orthogonal;
   (D) $\langle A, B \rangle = 3$, not orthogonal;
   (E) $\langle A,B \rangle = 0$, not orthogonal.

9. Which of the following matrices is the coordinate transformation matrix from a basis of R^2 consisting of $v_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ to another basis of R^2 consisting of $u_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$?
   (A) $\begin{bmatrix} 3 & 4 \\ -4 & -5 \end{bmatrix}$;
   (B) $\begin{bmatrix} -3 & -4 \\ 4 & -5 \end{bmatrix}$;
   (C) $\begin{bmatrix} 3 & 5 \\ 7 & 8 \end{bmatrix}$;
   (D) $\begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$;
   (E) $\begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$.

10. Which of the following statements is/are true?
   (A) Every nonzero finite-dimensional inner product space has an orthonormal basis.
   (B) Let A be an $m \times n$ matrix with rank n and $m \geq n$. If A^* is the adjoint matrix of A, then A^*A is invertible.
   (C) A periodic function is in an inner product space with infinite linearly independent vectors.
   (D) A square matrix that is diagonalizable must be full ranked.
   (E) Any normal operator in a finite-dimensional inner product space is diagonalizable.

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11. Suppose that the motion of a certain spring-mass system satisfies the differential equation $u''+du'+\frac{5}{4}u = A\cos(\omega t)$ and the initial conditions: $u(0) = 2, u'(0) = 3$, where d, A, and ω are real numbers. Which of the following statements are true?
(A) When $d = 0$ and $A = 0$, the natural frequency of this unforced system is 1 Hz.
(B) When $d = 1$, ω is near 1, and $A = 3$, the amplitude of the above forced response will be quite large.
(C) When $d = 1$, $\omega = 1$, and $A = 3$, the solution of the given initial problem is
$u(t) = \frac{22}{17}e^{-t/2}\cos t + \frac{14}{17}e^{-t/2}\sin t + \frac{12}{17}\cos t + \frac{48}{17}\sin t$.
(D) When $d = 1$, $\omega = 1$, and $A = 3$, the amplitude of the steady-state solution is $R = \frac{15}{17}$.
(E) As $\omega \to \infty$, the amplitude of the steady-state solution will approach to zero for $d = 1$ and $A = 3$.

12. Consider the differential equation: $2y''+y'+2y = g(t)$. Which of the following statements are true?
(A) Let $g(t)$ be defined as follows: $g(t) = 1$, for $5 \le t < 20$; $g(t) = 0$, for $0 \le t < 5$ or $t \ge 20$. Thus, $g(t)$ is a continuous function.
(B) The Laplace Transform of $g(t)$ is $[e^{-5s} - e^{-20s}]/s$.
(C) Assume that the initial conditions are $y(0) = 1$ and $y'(0) = 0$. The Laplace Transform of $y(t)$ is $H(s)[e^{-5s} - e^{-20s}]/s$, where $H(s) = \frac{1}{2s^2+s+2}$.
(D) If $g(t)$ is changed to a unit impulse function applied at $t = 0$ and the initial conditions are $y(0) = 0$ and $y'(0) = 0$. The corresponding impulse response is
$y(t) = -\frac{1}{2}\left(e^{-4t}\cos\frac{\sqrt{15}}{4}t + \frac{1}{\sqrt{15}}e^{-4t}\sin\frac{\sqrt{15}}{4}t\right)$.
(E) The steady-state of this impulse response will approach to zero.

13. Let $f(x)$ be a periodical function $f(x+4) = f(x)$ in which $f(x)$ is defined as follows: $f(x) = -x$ for $-2 \le x < 0$, and $f(x) = x$ for $0 \le x < 2$. The Fourier series of $f(x)$ is of the form $f(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left(a_m\cos\left(\frac{m\pi x}{2}\right) + b_m\sin\left(\frac{m\pi x}{2}\right)\right)$. Which of the following statements are true?
(A) $f(x)$ is a differentiable function.
(B) $a_0 = 2$.
(C) $a_1 = \frac{-8}{\pi}$.
(D) $a_4 = 0$.
(E) $b_4 = 0$.

14. Consider the following differential equation: $(3xy + y^2) + (x^2 + xy)y' = 0$. Which of the following statements are true?
(A) This differential equation is an exact differential equation.
(B) This differential equation is a separable differential equation.
(C) To solve this differential equation, there exists an integrating factor that is a function of x only.
(D) $\mu(x, y) = 1/\left(xy(2x+y)\right)$ is also an integrating factor for this differential equation.
(E) Solutions of this differential equation are given implicitly by $x^3y + 0.5x^2y^2 = c$, where c is a constant.

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15. Consider the following initial value problem: $y' = G(x, y)$, where $G(x, y) = \dfrac{3x^2 + 4x + 2}{2(y-1)}$.

Which of the following statements are true?

(A) $G(x,y)$ is analytic in R^2.

(B) This initial value problem is a separable differential equation.

(C) The solution of this problem is given implicitly by $y^2 - 2y = x^3 + 2x^2 + 2x + c$, where c is real.

(D) When $y(0) = -1$, this problem has a unique solution in some interval about $x = 0$.

(E) When $y(0) = 1$, this problem has a unique solution in some interval about $x = 0$.

16. Which of the following expressions is correct? Note: z is complex number.

(A) $\cos(iz) = \cosh(z)$

(B) $\sinh(z) = i\sin(z)$

(C) $e^{\ln(z)} = z$

(D) $|\sinh(z)|^2 = \cosh^2(x) - \cos^2(y)$, where $z = x + iy$

(E) $\cos(z) = \dfrac{e^{iz} + e^{-iz}}{2}$

17. Evaluate the integral $f = \oint_C (z-a)^n\, dz$, where both z and a are complex numbers, n is any integer and C is a circle of radius R centered at a and oriented anticlockwise. Which of the following expressions is correct?

(A) When $n = 1$, $f = 2\pi i$,

(B) When $n = -1$, $f = 2\pi i$,

(C) When $n = 1000$, $f = 2\pi i$,

(D) When $n = -20$, $f = 0$,

(E) When $n = 1$, $f = 0$.

18. Compute $\int_{-\infty}^{\infty} \dfrac{1}{x^4 + 1}\, dx$, x is a real number.

(A) 0

(B) $\frac{\pi}{2}$

(C) 1

(D) $\pi\frac{\sqrt{2}}{2}$

(E) None of the above is correct.

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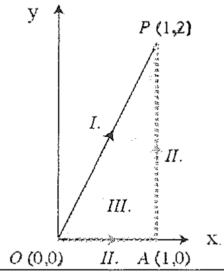
19. Evaluate the integral $f = \int_C z^2 dz$, where $z = x + iy$ is a complex number, in each of the following cases.

Paths:	
I. C is the straight line OP joining the points $O(0,0)$ and $P(1,2)$. II. C is the straight line from $O(0,0)$ to $A(1,0)$ and then from $A(1,0)$ to $P(1,2)$. III. C is the parabolic path $y = 2x^2$.	

Solution options:
a. $2\pi i$ b. $\frac{-1}{7}(12+5i)$ c. $\frac{-1}{3}(11+2i)$ d. 0

(A) Along all path I, II, and III, $f = a$.
(B) Along the path III, $f = b$.
(C) Along the path II, $f = c$.
(D) Along the path II and III, $f = a$, while along the path I, $f = d$.
(E) None of the above is correct.

20. Find all the Taylor and Laurent series expansions of $f(z) = \dfrac{1}{6 - z - z^2}$ with center 0. Which of the following expressions is correct?

Region		Expression	
I.	$\lvert z\rvert < 2$	a.	$\sum_{n=1}^{\infty}[\left(-\frac{2^{n-1}}{5}\right) + (\frac{(-1)^{n-1}3^{n-1}}{5})]\frac{1}{z^n}$
II.	$2 < \lvert z\rvert < 3$	b.	$\sum_{n=1}^{\infty}(-\frac{2^{n-1}}{5})\frac{1}{z^n} + \sum_{n=0}^{\infty}(\frac{(-1)^n}{5\times 3^{n+1}})z^n$
III.	$\lvert z\rvert > 3$	c.	$\sum_{n=0}^{\infty}\frac{1}{5}[\frac{1}{2^{n+1}} + \frac{(-1)^n}{3^{n+1}}]z^n$

(A) a. in region I. / b. in region II. / c. in region III.
(B) a. in region II. / b. in region I. / c. in region III.
(C) a. in region III. / b. in region I. / c. in region II.
(D) a. in region II. / b. in region III. / c. in region I.
(E) None of the above is correct