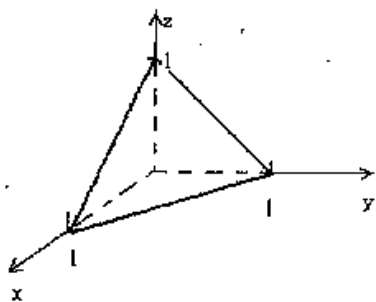


# 國立中央大學九十學年度碩士班研究生入學試題卷

所別: 大氣物理研究所 不分組 科目: 應用數學 共 1 頁 第 1 頁

1. Evaluate a surface integral of a vector function  $\vec{F}$  over  $S$ , i.e.,  $\iint_S \vec{F} \cdot \hat{n} dA$ ,

when  $\vec{F} = x^2\hat{i} + 3y^2\hat{k}$  and  $S$  is the portion of the plane  $x + y + z = 1$  in the first octant as shown in following figure.



(15%)

2.  $z$  is a complex variable, derive the trigonometric functions as

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

(15%)

3. In Cartesian coordinates  $x, y, z$  in space, the Laplacian of  $A$  is defined as

$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

Derive the Laplacian of  $A$  in the spherical coordinate.

(15%)

4. Find a general solution of the following equation

$$x^2 y'' - 3xy' + 4y = 12$$

(10%)

5. Find the eigenvalues and eigenfunctions of the following problem

$$y'' + \lambda y = 0, \quad y(0) = y(2L), \quad y'(0) = y'(2L)$$

(15%)

6. Solve the following initial value problem by means of Laplace transforms

$$\begin{aligned} y_1' &= -y_1 + y_2, & y_1(0) &= 1, & y_2(0) &= 0 \\ y_2' &= -y_1 - y_2 \end{aligned}$$

(15%)

7. Find the eigenvalues and eigenvectors of the following matrix

$$\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

(15%)

