

1. What is the Laplace transform? Please use 2 different methods (one of which must be the Laplace transform) to solve the following equations,

(i) $y'' + y = 3 \cos 2t$, $y(0) = 0$, $y'(0) = 0$

and

(ii) $y'' - 2.8y' + 1.96y = 2e^{1.4t}$ $y(0) = 0$, $y'(0) = 0$.

(20%)

2. Please solve the equation,

$$y'' + y = F \cos(t) \quad \text{with } y(0) = A, \quad y'(0) = 0,$$

and discuss its solution characteristics. What is resonance? Can you describe resonance physically with this equation and its associated solution?

(15%)

3. Use the Fourier-Legendre expansion method to represent the following functions,

(a) $f(x) = 15 - 42x^2 + 35x^4$

and

(b) x^3 .

Hint: $p_0 = 1$, $p_1 = x$, $p_2 = \frac{1}{2}(3x^2 - 1)$, $p_3 = \frac{1}{2}(5x^3 - 3x)$ and

$$p_4 = \frac{1}{8}(35x^4 - 30x^2 + 3).$$

(15%)

4. Define gradient, divergence and curl. Show some of their applications in physics and state their physical meaning.

(10%)

5. What is Green's theorem? What is divergence theorem? What is Stokes's Theorem? When and where you can apply these theorems? Show some examples.

(10%)

6. Find the eigenvalues and eigenvectors of the following matrix,

$$\begin{bmatrix} 0 & 2 & 0 \\ 3 & -2 & 3 \\ 0 & 3 & 0 \end{bmatrix}.$$

(15%)

7. Find the potential in the square $0 \leq x \leq 2$, $0 \leq y \leq 2$ if the upper side is kept at the potential $\sin \frac{1}{2}\pi x$ and the other sides are kept at zero.