

國立中央大學97學年度碩士班考試入學試題卷

所別：水文與海洋科學研究所碩士班

科目：應用數學 共 1 頁 第 1 頁

\*請在試卷答案卷(卡)內作答

1. Calculate the curl of each of the following vector functions  $\mathbf{F}$ :

(a) [5%]  $\mathbf{F} = \hat{i}z^2 + \hat{j}x^2 - \hat{k}y^2$

(b) [5%]  $\mathbf{F} = (\hat{i}x + \hat{j}y + \hat{k}z) / (x^2 + y^2 + z^2)^{3/2}, \quad (x, y, z) \neq (0, 0, 0)$

2. Consider the surface  $S$  defined by

$$\begin{aligned} x &= (a + b \cos \phi) \cos \theta \\ y &= (a + b \cos \phi) \sin \theta \\ z &= b \sin \phi \end{aligned} \quad \text{for} \quad \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq \phi \leq \pi \end{cases}$$

where  $a$  and  $b$  are constants such that  $0 < b < a$ .

(a) [10%] Find the two tangent vector to the surface,  $\bar{b}_\theta$  and  $\bar{b}_\phi$ ,

in terms of  $\theta$  and  $\phi$ .

(b) [10%] Find the unit normal vector  $\hat{n}$  to the surface  $S$ .

(c) [10%] Calculate the value of the surface integral  $\iint_S \bar{\mathbf{F}} \cdot \hat{n} dA$

for the vector field  $\bar{\mathbf{F}} = z\hat{k}$ .

3. Solve the following ordinary differential equations for  $y = y(x)$ :

(a) [10%]  $(1 + x^2) \frac{dy}{dx} + 4xy = x$ , and  $y(1) = \frac{1}{4}$ .

(b) [10%]  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2y = 0$ , and  $y(1) = 0, \frac{dy(1)}{dx} = 1$ .

4. [20%] Use the method of variation of parameter to find the general

solution  $y(x)$  for  $\frac{d^2y}{dx^2} + y = \sec x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$ .

5. [20%] For the matrix  $\mathbf{A} = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$ , find the eigenvalues and the

corresponding eigenvectors of the matrix  $\mathbf{A}$ .

參考用