國立中央大學八十九學年度碩士班研究生入學試題卷

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參考用

1. (20%) Justify the formula of Euler:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

2. (20%) Please use the principle of mathematical induction to show that $\left(1+\frac{1}{n}\right)^n > 2$ for all $n \geq 2$.

3. (15%) Each f_n is a polynomial and $\{f_n\}$ converges uniformly to f on $[0,1]$. Prove or disprove that f is a polynomial.

4. (15%) Let $\{x_n\}$ be a convergent sequence in a metric space and $\lim_{n\to\infty} x_n = x$. Prove or disprove that the set $A = \{x_1, x_2, x_3, \cdots\} \cup \{x\}$ is compact?

5. (15%) Define

$$f(x) = \left(\int_0^x e^{-t^2}\, dt\right)^2, \quad g(x) = \int_0^1 \frac{e^{-x^2(t^2+1)}}{t^2+1}\, dt.$$

Show that $f(x) + g(x) = \frac{\pi}{4}$ for all $x \in \mathbb{R}$.

6. (15%) Given $\sum_{n=1}^{\infty} c_n$, $c_n \in \mathbb{R}$, put $\alpha = \limsup_{n\to\infty} \sqrt[n]{|c_n|}$. Prove that if $\alpha > 1$, then the series $\sum_{n=1}^{\infty} c_n$ diverges.