1. (15%) Let V and W be finite dimesional vector spaces of dimensions m and n respectly. Prove or disprove that for any given basis $v_1, v_2, \ldots, v_m$ of V and any given m vectors $w_1, w_2, \ldots, w_m$ of W there is one and only one linear transformation T from V to W such that $T(v_i) = w_i$ for all $i = 1, 2, \ldots, m$.

2. (15%) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{bmatrix}$, where $i^2 = -1$. Find the minmal polynomial of A and justify your claim.

3. (15%) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. Find an unitary matrix U such that $U^{-1}AU$ is diagonal and a diagonal matrix simiiar to A.

4. (15%) Show that the set of all functions $f(t)$ satisfying the differential equation
$$\frac{d^2 f}{d^2 t} - 5\frac{df}{dt} + 6f = 0$$
is 2-dimensional vector space with basis $\{e^{2t},\ e^{3t}\}$.

5. (20%) Find the inverses of the following matrices if they are invertible.

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & -1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 2 & 2 \\ 3 & 2 & 1 & 0 & 1 \\ 1 & -1 & 4 & 0 & 1 \\ 4 & 2 & 1 & 2 & 1 \\ -3 & 2 & 1 & 0 & 5 \end{bmatrix}$$

6. (10%) Find the eigen values of the matrix $\begin{bmatrix} 1 & -3 & 3 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

7. (10%) If T is a linear transformation from a finite dimensinal vector space V to itself such that $T^2 - 2T + I = 0$, show that there is a $v \neq 0$ in V such that $T(v) = v$.