國立中央大學九十學年度碩士班研究生入學試題卷

所別: 數學系 不分組　科目: 抽象代數　共 1 頁 第 1 頁

一. Show that if every element of a group G is its own inverse, then G is abelian (or commutative). (10%)

二. Let G be the group of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $ad-bc\neq 0$ and a, b, c, d are integers modulo 3, relative to matrix multiplication. Show that the order of G is 48. (10%)

三. If a cyclic subgroup T of G is normal in G, prove that every subgroup of T is normal in G. (10%)

四. If G is of order 108, show that G has a normal subgroup of order 3^k where $k\geq 2$. (10%)

五. Prove that a finite integral domain is a field. (10%)

六. Let ϕ be a ring-homomorphism from R onto R' and let W' be an ideal in R'. If $W=\{x\in R \mid \phi(x)\in W'\}$, prove that W is an ideal in R and R/W is isomorphic to R'/W'. (5%; 10%)

七. Let R be a commutative ring and let A be an ideal of R. The radical $N(A)=\{x\in R \mid x^n\in A$ for some positive integer $n\}$. Prove

(a) $N(A)$ is an ideal of R which contains A. (5%)

(b) $N(N(A))=N(A)$. (10%)

八. Let F be a field and let $f(x)\in F[x]$ be an irreducible polynomial. If the characteristic of F is 0, prove that $f(x)$ has no multiple roots. (10%)

九. Find the Galois group of x^3-3x-3 over the rational field $\mathbb{Q}$. (10%)