

系所別:

數學系

科目:

離散數學

1. (20%) A magic square of order n is an n -by- n array constructed out of the integers $1, 2, 3, \dots, n^2$ in such a way that the sum of the integers in each row, in each column, and in each of the two diagonals is the same number. Show that the result of replacing every integer k in a magic square of order n with $n^2 + 1 - k$ is a magic square of order n .
2. (20%) Show that for any given 52 integers there exist two of them whose sum, or else whose difference, is divisible by 100.
3. (20%) For all non-negative integers k and n , let
- $$\binom{n}{k} = \begin{cases} 1, & k = 0 \\ \frac{n!}{k!(n-k)!}, & 1 \leq k \leq n \\ 0, & k > n. \end{cases}$$
- Prove that $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$, ($n \geq 0$).
4. (20%) Determine the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ which satisfy $1 \leq x_1 \leq 5$, $-2 \leq x_2 \leq 4$, $0 \leq x_3 \leq 5$, $3 \leq x_4 \leq 9$.
5. (20%) Prove that A simple graph of order $n \geq 3$, in which the sum of the degrees of each pair of nonadjacent vertices is at least n , has a Hamiltonian cycle.

參考用