國立中央大學97學年度碩士班考試入學試題卷

所別：機械工程學系碩士班 甲、乙、丙組　　科目：工程數學　共 2 頁 第 1 頁

＊請在試卷答案卷（卡）內作答

能源工程研究所碩士班

光機電工程研究所碩士班 乙組(光機組)

生物醫學工程研究所碩士班

Ordinary Differential Equation (25 %)

1. Find the family of the curve such that the projection on the $x-$axis of the part of the tangent between (x,y) and the x axis has length 1. (5%)

2. A 6 *lb.* weight is attached to the lower end of a spring suspended from the ceiling, the spring constant being $27lb/ft$. The weight comes to rest, and beginning at $t=0$ an external force given by $F(t)=12\cos 20t$ is applied to the system. Determine the resulting displacement as a function of time, assuming damping is negligible. (10%)

3. How many methods can you use to solve the differential equation

$$2xydx+(y^2-x^2)dy=0$$

Explain your answers. (10%)

參考用

Linear Algebra & Vector Calculus (25 %)

4. Show that the differential form under the integral sign of

$$I=\int_{(-1,5)}^{(4,3)}(3z^2dx+6xzdz)$$

is exact, so that we have independence of path in any domain, and find the value of the integral I from A: (-1, 5) to B: (4, 3). (10%)

5. Find out what type of conic section is represented by the given quadratic form.

$Q = 17\ x_1^2 - 30\ x_1x_2 + 17\ x_2^2 = 128$. Transform it to principal axes. (15%)

注意:背面有試題

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Complex Analysis (25 %)

6. Determine where the function, $f(z) = 2x - x^3 - xy^2 + i(x^2 + y^3 - 2y)$, is analytic. (10%)

7. Evaluate the following integral counterclockwise. (15%)

$$\oint_C \cot\frac{z}{4}dz, \qquad C: |z| = 1.$$

Partial Differential Equation and Fourier Analysis (25 %)

8. Show that the Fourier series of $f(x) = x$, $-\pi < x < \pi$ leads to

$$\frac{\pi}{4} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots \qquad (10\%)$$

9. Solve the partial differential equation (15%)

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - 1, \quad 0 < x < 1, \quad t > 0$$

$$T(x,0) = \frac{x^2}{2} + \cos(\pi x), \quad 0 < x < 1$$

$$\frac{\partial T(0,t)}{\partial x} = 0, \quad \frac{\partial T(1,t)}{\partial x} = 1, \quad t > 0.$$

注意:背面有試題