所別：機械工程學系碩士班 甲組(固力與設計) 科目：工程數學
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一、(33%=5%+5%+10%+5%+8%)

1.

(a) Solve the initial value problem $y' + y^2 = 1,\ y(0) = 0$. (5%)

(b) Find the general solution of the differential equation (5%)

$$\frac{dy}{dx} = \frac{-y\cos x - \sin y}{x\cos y + \sin x}$$

2. Find the general solution of the differential equation (10%)

$$y'' + y' - 2y = (x+1)e^x$$

3. Consider the eigenvalue problem

$$\left(p(x)y'\right)' + \left\{q(x) + \lambda r(x)\right\} y = 0,\quad \alpha < x < \beta$$
$$p(\alpha) = p(\beta),\ y(\alpha) = y(\beta),\ y'(\alpha) = y'(\beta)$$

where $p \geq 0$, $q \leq 0$, and $r > 0$ are all real and continuous for $\alpha \leq x \leq \beta$.

(a) Show that eigenfunctions corresponding to distinct eigenvalues are orthogonal with respect to the weight function $r(x)$. (5%)

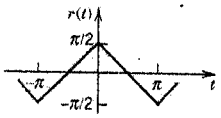
(b) Show that all eigenvalues are real and nonnegative. (8%)

二、(33%=9%+9%+9%+6%)

1. Evaluate the surface integral $\iint_S \vec{F} \bullet \vec{n} dA$,

where $\vec{F} = -x\cos(xy)\vec{i} + y\cos(xy)\vec{j} + (4x^2 + 5xy + 6z\vec{k}$ and S is the surface of the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$. (9%)

2. Find the Fourier series of the periodic function $r(t)$ of period $p = 2\pi$, as shown below. (9%)



3. Mathematically prove that the eigenvalues of a symmetric matrix are real. (9%)

4. It has been known that $curl(\nabla f)$ is a zero vector for any twice continuously differentiable scalar function f. Physically explain why? (6%)

注意：背面有試題

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三、(34% = 10%+12%+12%)

1. Use separation of variables to find product solutions of $\frac{\partial^2 u}{\partial x \partial y} = u$. (10%)

2. Use the Laplace transform to solve

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < 3, \ t > 0$$

$$u(0,t) = 0, \quad u(3,t) = 0, \ t > 0$$

$$u(x,0) = \sin\frac{\pi x}{3}, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0 \qquad (12\%)$$

3. Evaluate $\int_0^{\pi} \frac{\cos 2\theta}{2 - \cos\theta} d\theta$. (12%)