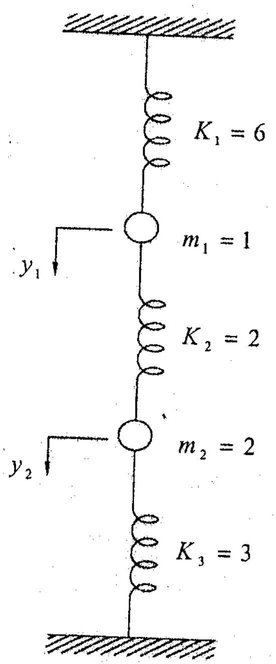
國立中央大學94學年度碩士班考試入學試題卷　共 2 頁 第 1 頁

所別：能源工程研究所碩士班 科目：工程數學

Ordinary Differential Equation (33 %)



1. Consider the mass-spring system as shown in the figure. Assume that there is no damping and that no external force is applied to the system. Suppose that the upper weight is pulled down two units and the lower weight is raised one unit, then both weights are released from rest simultaneously at time $t = 0$.
   (1) Please derive the system of two second order differential equations governing the position of the weights relative to their equilibrium positions at any time $t = 0$. Note that only the system of differential equation is required. (5%)
   (2) Please convert the system of two second order differential equations, you have obtained in (1), into a system of four first order differential equations. (5%)
   (3) Let the system of four first order differential equations be written as $X' = AX$. Determine A and $X(0)$? (5%)
   (4) Determine the eigenvalues of A. (5%)

2. Suppose that the differential equation $P(x,y)dx + Q(x,y)dy = 0$ is not exact.
   (1) What is the necessary condition for the differential equation to have an integrating factor $F = F(x)$. (4%)
   (2) Let $P(x,y) = -y$ and $Q(x,y) = x$. Determine an integrating factor $F = F(y)$ of the differential equation. (4%)

3. Given

$$J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu} m!\Gamma(\nu + m + 1)}$$

where $J_\nu(x)$ is known to be the Bessel function of the first kind of order ν, and Γ the Gamma function. Show that

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x. \qquad (5\%)$$

Note that you may directly use $\Gamma(1/2) = \sqrt{\pi}$ without proof.

注意：背面有試題

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Linear Algebra & Vector Calculus (33 %)

4. Using Green's theorem, evaluate the line integral $\oint_C \mathbf{F}(\mathbf{r})d\mathbf{r}$ counterclockwise around the boundary C of a closed region R, where

   $\mathbf{F} = [x\cosh(2x), x^2\sinh(2y)]$, R: $x^2 \le y \le x$. (10%)

5. Evaluate surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}dA$ for the following data:

   $\mathbf{F} = [6x^2, 4y^2, 0]$ S: $\mathbf{r} = [u, v, 3u+6v]$, $1 \le u \le 2$, $-2 \le v \le 2$. (15%)

6. Let $\mathbf{F} = \begin{bmatrix} 2 & 2 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, find a symmetric matrix **B** and a skew-symmetric matrix **C**, such that

   $\mathbf{B} + \mathbf{C} = \mathbf{A}$. (8%)

Fourier Analysis, Partial Differential Equation and Complex Analysis (34 %)

7. (a) Expand $f(x) = x + \pi$, $-\pi < x < \pi$ in a Fourier series. (7%)

   (b) Use the result of (a) to find $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$ (3%)

8. (a) Solve the wave equation

   $$a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2},\ 0 < x < L,\ t > 0$$

   $$u(0, t) = 0,\ u(L, t) = 0,\ t > 0$$

   $$u(x, 0) = f(x),\ \frac{\partial u}{\partial t}\Big|_{t=0} = 0,\ 0 < x < L \text{ (5\%)}$$

   (b) Show that the solution of (a) can be written as $u(x,t) = \frac{1}{2}[f(x+at) + f(x-at)]$. (5%)

9. (a) Expand $f(z) = \dfrac{1}{z(z-1)}$ in a Laurent series that is valid in a deleted neighborhood of $z = 1$.

   State the domain throughout which the series is valid. (3%)

   (b) Find $\oint_C \dfrac{1}{z(z-1)}dz$, where C is the circle $|z-1| = 6$, by means of the residue theorem. (3%)

   (c) Find $\int_0^\infty \dfrac{dx}{x^6+1}$. (8%)