

系所別： 土木工程學系 己組 科目： 統計學

註1：共5題，一百分。

註2： $Z_{0.95} = 1.645$, $Z_{0.975} = 1.96$, $Z_{0.99} = 2.326$, $Z_{0.995} = 2.575$ 。

1. (25%) A biased four-sided die is rolled once. Random variable N is defined to be the down-face value and is described by the PMF,

$$P_N(N_0) = \begin{cases} N_0/10 & \text{for } N_0 = 1, 2, 3, 4 \\ = 0 & \text{for all other values of } N_0 \end{cases}$$

Based on the outcome of this roll, a coin is supplied for which, on any flip, the probability of an outcome of heads is $(N+1)/2N$. The coin is flipped once, and the outcome of this flip completes the experiment.

Determine:

- (a) (8%) The expected value and variance of discrete random variable N .
- (b) (10%) The conditional PMF, conditional expected value and conditional variance for random variable N given that the coin came up heads.
- (c) (7%) If we define the events,
Event A: Value of down face on die roll is either 1 or 4.
Event H: Outcome of coin flip is heads.
Are the events A and H independent?
2. (25%) The PDF for the duration of the independent inter-arrival times between successive cars on a highway is given by

$$f_t(t_0) = \begin{cases} \frac{1}{12} e^{-t_0/12} & t_0 \geq 0 \\ = 0 & t_0 \leq 0 \end{cases}$$

Where these duration are measured in seconds.

- (a) (8%) An old wombat requires 12 seconds to cross the highway, and he starts out immediately after a car goes by. What is the probability that he will survive?
- (b) (7%) Another old wombat, slower but tougher, requires 24 seconds to cross the road, but it takes two cars to kill him. (A single care won't even slow him down.) If he starts out at a random time, determine the probability that he survives.
- (c) (10%) If both these wombats leave at the same time, immediately after a car goes by, what is the probability that exactly one of them survives?
3. (15%) 交通部委託研究機構作一項市場調查，在抽得的 225 個女性中有 105 個喜歡開快車，275 個男性中有 145 個喜歡開快車，試以 $\alpha=0.05$ 檢定所有男、女喜歡開快車的比例是否相等？

參考用

注意：背面有試題

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4. (15%) 一汽車工廠生產的汽車,其使用月份的標準差 $\sigma=15$ 個月,今隨機抽查 25 輛汽車,得平均使用月份是 720 個月,試以 (1) $1-\alpha=0.95$, (2) $1-\alpha=0.99$ 為信賴度,試求算所有汽車平均使用月份 μ 的信賴區間及信賴區間長度 w 。
5. (20%) 一房地產投資公司為研究某住宅戶建築用地的合理價格,抽訪了此地區最近成交的五筆住宅戶建築用地買主或賣主,這五筆土地的坪數、公告地價或成交價格如下:

土地坪數 X_1 (百坪)	公告地價 X_2 (百萬元)	成交價格 Y (百萬元)
9	8	36
15	7	80
10	9	44
11	10	55
10	6	35

此等參數的部分統計如下表所示:

i	X_{1i}	X_{2i}	Y_i	$X_{1i} * X_{1i}$	$X_{2i} * X_{2i}$	$X_{1i} * X_{2i}$	$X_{1i} * Y_i$	$X_{2i} * Y_i$
1	9	8	36	81	64	72	324	288
2	15	7	80	225	49	105	1200	560
3	10	9	44	100	81	90	440	396
4	11	10	55	121	100	110	605	550
5	10	6	35	100	36	60	350	210
total	55	40	250	627	330	437	2919	2004

- (a) (10%) 試求得估計線性迴歸方程式。
- (b) (10%) 試計算說明(a)所求的線性迴歸方程式,其線性迴歸關係是否成立。

參考用