

國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 產業經濟研究所 甲組 科目:

微積分

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1. Find the solutions to the following differential equations, subject to the given initial conditions.

(a) $\frac{dy}{dt} = 0.05(y - 4000)$, $y(0) = 3000$. (10%)

(b) $(\cos x - x \sin x + y^2) dx + 2xy dy = 0$, $y(\pi) = 1$. (10%)

2. For every real number x , let $g(x) = \cos(e^{\cos x})$. Find $g'(x)$. (15%)

3. Prove or disprove that $e^x \geq e^\pi x + e^\pi(1 - \pi)$ for every real number x . (20%)

4. Prove or disprove that $\lim_{x \rightarrow 0} \frac{\sin x - (\cos x)(\sin x)}{x^2} = 0$. (15%)

5. Suppose the total cost of producing q goods is $C(q) = 0.01q^3 - 0.6q^2 + 13q$. What is the maximum profit if each item is sold at a price of \$7? (15%)

(Assume you can sell everything you produce.)

6. Let $P(t) = 0.0000001(t^2 - 0.001339t^4 + 123450)$ be a model for the density function for the US age distribution, where t is in years between 0 and 100. Using this density function to find the mean age of the US population. (15%)

