

# 國立中央大學九十一年度碩士班研究生入學試題卷

所別: 產業經濟研究所 甲組 科目: 統計學 共 / 頁 第 / 頁

1. A single observation  $X$  is distributed uniformly on the interval  $[0, \theta]$ ,  $\theta > 0$ . Calculate the risk function for the decision function  $d(x) = cx^2$  when the loss function is quadratic,  $L(\theta, a) = (\theta - a)^2$ . (20%)
2.  $X_1, X_2, \dots, X_n$  is a random sample and  $X_1$  has a density of the form  $g(x_1 | \theta) = \theta^2 x_1 e^{-\theta x_1}$ ,  $x_1 > 0$ , (= 0 elsewhere),  $\theta > 0$ . (30%)
- Find the maximum likelihood estimator for  $\theta$ .
  - Find the Cramer-Rao <sub>lower</sub> bound for the variance of unbiased estimators of  $\lambda(\theta) = \theta^2$ .
  - Find the method of moments estimator of  $\theta$ .
3. Consider a sequential probability ratio test which stops sampling as soon as the value of a likelihood ratio falls outside the interval  $(\frac{1}{20}, 10)$ . Approximately, what are the values of the error probabilities  $\alpha$  and  $\beta$ ? (20%)
4. Let  $X_1$  and  $X_2$  be independent Bernoulli random variables with unknown common means equal to  $p \in [0, 1]$ , and let  $T = X_1 + X_2$ . (30%)
- Show that  $T$  is a sufficient statistic for  $p$  or not.
  - In order to show that  $T$  is a complete statistic for  $p$ , what, precisely, must be shown? (There is no need to show that  $T$  is complete.)

參考用