1. A single observation X is distributed uniformly on the interval $[0, \theta]$, $\theta>0$. Calculate the risk function for the decision function $d(X)=cX^2$ when the loss function is quadratic, $L(\theta, a)=(\theta-a)^2$. (15%)

2. $X_1, X_2, \ldots, X_n$ is a random sample and X_1 has a density of the form $g(X_1 \mid \theta)= \theta^2 X_1 e^{-\theta X_1}$, $X_1 \geq 0$ (=0 elsewhere), $\theta>0$.
   a) Find the maximum likelihood estimator for θ. (15%)
   b) Find the Cramer-Rao bound for the variance of unbiased estimator of $\lambda(\theta)=\theta^2$. (15%)
   c) Find the method of moment estimator of θ. (15%)

3. Consider the following two-variable model:

Model I: $Y_i=\beta_1+\beta_2 X_i+\mu_i$
Model II: $Y_i=\alpha_1+\alpha_2(X_i-\overline{X})+\mu_i$

a). Find the estimators of β_1 and α_1. Are they identical? Are their variances identical? (10%)
b). Find the estimators of β_2 and α_2. Are they identical? Are their variances identical? (10%)

4. Consider the following models:

$$\ln Y_i^* = \alpha_1 + \alpha_2 \ln X_i^* + \mu_i^*$$

$$\ln Y_i = \beta_1 + \beta_2 \ln X_i + \mu_i$$

Where $Y_i^*=w_1 Y_i$ and $X_i^*=w_2 X_i$, the w's being constants.
a). Establish the relationships between the two sets of regression coefficients and their standard errors. (10%)
b). Is the r^2 different between the two models? (10%)