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1. (10%) Consider the sinusoidal pulse $x(t) = \Pi\left(\frac{t}{8T_0}\right)\sin(2\pi f_0 t)$, where $f_0 = 1/T_0$.
- a) Find the Fourier transform of $x(t)$.
 - b) Is $x(t)$ bandlimited? Find the minimum sampling frequency that can perfectly reconstruct $x(t)$ from its samples.
2. (20%) A message signal $m(t) = \cos(2\pi f_m t)$, $0 \leq f_m \leq W$, is modulated by an amplitude modulation system where the modulated signal is $x_c(t) = (A + m(t))\cos\omega_c t$, where A is a constant.
- A coherent demodulator, in which a carrier $\cos\omega_c t$ is multiplied with the received signal and a lowpass filter with the bandwidth W is placed, is used to recover $m(t)$.
- a) Case 1: the message signal is first passed through a limiter, i.e., $m(t)$ becomes a square wave with the fundamental frequency f_m before modulation. Find the range of f_m that the original sinusoidal signal can be fully recovered after demodulation. Assume the original demodulator is used.
 - b) Case 2: the carrier is saturated before modulation, i.e., a square pulse train with the fundamental frequency ω_c instead of the sinusoidal carrier is used to modulate the message. Plot the spectrum of the modulated signal. Describe the filter specification that is needed in the modulation system to avoid the interference to other channels.
3. (20%) Let the sample function of a random process be given by $X(t) = A\cos(2\pi f_0 t + \Theta)$, where f_0 and Θ are constants, and A is a random variable with the pdf $f_A(a) = \frac{e^{-a^2/2\sigma_a^2}}{\sqrt{2\pi}\sigma_a}$. This random process $X(t)$ is passed through a filter with the frequency response $H(f) = f$ to generate a random process $Y(t)$.
- a) Find an expression for the sample functions of the output process $Y(t)$.
 - b) Write down an expression for the pdf of $Y(t)$ at time t_0 .
 - c) Find the variance of $Y(t)$ at time t_0 .
 - d) Find the power, i.e., the second moment time-average, of $Y(t)$.

注意：背面有試題

4. (15 %) A communication system transmits one of three real-valued signals,

$$s_0(t) = \sqrt{P} p_T(t), s_1(t) = 0, \text{ and } s_2(t) = -\sqrt{P} p_T(t), \text{ every } T \text{ seconds, where } p_T(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases}$$

The received signal is either $r(t) = s_0(t) + z(t)$, $r(t) = s_1(t) + z(t)$, or $r(t) = s_2(t) + z(t)$, where $z(t)$ is white Gaussian noise with $E[z(t)] = 0$ and $E\{z(t)z(\tau)\} = \frac{1}{2} N_0 \delta(t - \tau)$. The optimum receiver computes the correlation metric

$$U = \sqrt{P} \int_0^T r(t) p_T(t) dt$$

and compares U with a threshold A and a threshold $-A$. If $U > A$, the decision is made that $s_0(t)$ was sent. If $U < -A$, the decision is made in favor of $s_2(t)$. If $-A < U < A$, the decision is made in favor of $s_1(t)$.

a) Determine the three conditional probabilities of error using Q function, A , E , T , and N_0 : $P_{e,0}$

given that $s_0(t)$ was sent, $P_{e,1}$ given that $s_1(t)$ was sent, and $P_{e,2}$ given that $s_2(t)$ was sent.

$$\text{(Note: } Q(v) = \int_v^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy \text{)}$$

b) Determine the average probability of error P_e , assuming that the three symbols are equally probable a priori.

5. (15 %) Let $y = s + n$ where n is exponential with probability density function $f(n) = \frac{1}{10} e^{-n/10}$ if $n \geq 0$, and $f(n) = 0$ otherwise. Suppose hypothesis, H_1 is $s = 1$ and hypothesis, H_0 is $s = -1$.

$f(y|s) = \frac{1}{10} e^{-\frac{(y-s)}{10}}$ if $y \geq s$, and $f(y) = 0$ otherwise. Suppose $p(s = 1) = p(s = -1) = \frac{1}{2}$. Find the Bayes detector for H_0 and H_1 . That is find the regions for y where we detect H_0 , H_1 and where there is no detection.

6. (5 %) A coherent M-ary FSK signal system employ 16 signals. What channel bandwidth would be required if the message bit rate is 2000 bps? Please provide your explanation for your answer.

7. (15 %) Let $X \in \{1, 2, 3, 4\}$ and $Y \in \{1, 2, 3, 4\}$ be two discrete random variables.

Let (X, Y) have the following joint distribution:

Y	X	1	2	3	4
1		1/8	1/16	1/16	1/4
2		1/16	1/8	1/16	0
3		1/32	1/32	1/16	0
4		1/32	1/32	1/16	0

Determine $H(X)$, $H(X, Y)$, $I(X; Y)$, and $H(Y|X)$.