

所別：通訊工程學系碩士班 甲組(一般生) 科目：工程數學

1. (20%) Let $i^2 = -1$. Find

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{1}{\sqrt{s}} e^{st} ds, \text{ where the integration contour is parallel to the Im } s\text{-axis with } c > 0.$$

2. (15%) Let $y' = \frac{dy}{dx}$. Please solve

$$yy'' - (y')^2 = y^2 \ln y - x^2 y^2.$$

3. (15%) Find

(7%) (a). The unit normal vector of the surface $z^2 = xy$ at the point $(1, 2, 2)$.

(8%) (b). The tangent plane of the surface $x^2 = 1 - (y^2 + z^2)$ at the point $(1, 1, 1)$.

4. (20%) Please find the expectation (mean) $E[X]$ of following random variables X :

(i) Binomial random variable: $p(x) = P\{X = x\} = \binom{n}{x} p^x \cdot (1-p)^{n-x}$, $x = 0, 1, 2, \dots, n$

(ii) Geometric random variable: $p(x) = P\{X = x\} = p \cdot (1-p)^{x-1}$, $x = 1, 2, \dots, \infty$

(iii) Exponential random variable: $f(x) = \lambda e^{-\lambda x} u(x)$

(iv) Normal (Gaussian) random variable: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

5. (10%) The joint probability density function of random variables X and Y is given by

$$f_{XY}(x, y) = \begin{cases} \frac{1}{2} ye^{-xy} & 0 < x < \infty, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Please find $E[e^{X/4} | Y = 1]$.

6. (20%) Consider independent and identical Gaussian random variables $X_i, i = 1, 2, \dots, M$, with mean and variance equal to 0 and 1 respectively. Please find

(i) The probability density function $f_Y(y)$ of $Y = X_1 + X_2 + \dots + X_M$.

(ii) The probability density function $f_Z(z)$ of $Z = \sqrt{X_1^2 + X_2^2}$.