

# 國立中央大學九十一年度碩士班研究生入學試題卷

所別: 通訊工程研究所 甲乙組 科目: 工程數學 共 1 頁 第 1 頁

10% 1. Please find the output signal  $y(t)$  of a circuit system, which is described by the linear differential equation  $\frac{dy(t)}{dt} + y(t) = x(t)$  with initial condition  $y(0) = 0$  and the input signal  $x(t) = 2 \sin \omega t \cdot u(t)$ .

20% 2.

(a) Please find  $A^{40}$  with  $A = \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix}$ .

(b) Please show the following determinant

$$\begin{vmatrix} 1 & a_1 & a_1^2 & \cdots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & \cdots & a_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & a_n & a_n^2 & \cdots & a_n^{n-1} \end{vmatrix} \neq 0 \quad \text{where } a_1 \neq a_2 \neq a_3 \cdots \neq a_{n-1} \neq a_n \neq 0.$$

20% 3. The Fourier Transform pair of a signal are defined as

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad \text{and} \quad f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

Please find the Fourier Transform  $F(\omega)$  of the following signals

(a)  $f(t) = \frac{1}{a + t^2}$       (b)  $f(t) = \frac{\sin t}{t}$

10% 4. If  $K$  is a discrete random variable with the probability mass function

$$P(K = k) = \begin{cases} C \binom{4}{k}, & k = 0, 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases} \quad \text{Find the constant } C.$$

20% 5. The random variables  $X$  and  $Y$  are uniformly distributed over a circle of unit radius with the probability density function

$$f(x, y) = \begin{cases} C, & \text{if } 0 < x^2 + y^2 < 1 \\ 0, & \text{otherwise} \end{cases}$$

5% (i) Find the constant  $C$ .

15% (ii) Find the probability  $P(|X - Y| > 1)$ .

20% 6. A pressure control apparatus contains 1000 electronic tubes. The apparatus will work if either one of the tubes is operative. The probability of failure of each tube during some interval of time is 0.004.

15% (i) What is the corresponding probability that the apparatus will work?

5% (ii) If the apparatus is cascaded to a similar electronic tube, what is the corresponding probability that the apparatus will work?

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