

系所別: 通訊工程學系 甲組 科目: 工程數學
乙組



10% 1. Let $\{x, y, z\}$ be a basis for a vector space V . Please show that $\{x+y, y, x+y+z\}$ is also a basis for V .

12% 2. Please compute the inverse of (a) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & -3 \\ 3 & 1 & 0 \\ -6 & -4 & 2 \end{bmatrix}$.

12% 3. Let A and B be unitary matrices. Please show that the inverse of the matrix $C=AB$ is also unitary.

16% 4. Find a matrix Q such that $Q^{-1}AQ$ is a diagonal matrix for $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{bmatrix}$.

15% 5. The moment generating function $G(t)$ of a discrete random variable X is defined as $G(t) = E(e^{tX}) = \sum_j e^{tx_j} p(x_j)$,

5% (a) please find the moment generating function $G(t)$ of the Poisson distribution given by $p(x) = \frac{a^x}{x!} e^{-a}$ $x = 0, 1, 2, \dots$.

10% (b) please use the moment generating function $G(t)$ to find the mean μ and variance σ^2 of the Poisson distribution.

20% 6. The probability density function of a normal (Gaussian) random variable X

$$\text{is given by } f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}},$$

5% (a) please compute $P(X \leq 5 | X > 1)$ in terms of Q -function with the

$$Q\text{-function defined as } Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz.$$

5% (b) please find the mean $E(Y)$ of the random variable $Y = 5X^2 + 10$.

10% (c) please find the probability density function $f_z(z)$ of the random variable $Z = 2X + 5$.

15% 7. Consider two independent random variables X and Y with probability

density function given respectively by $f_x(x) = \alpha e^{-\alpha x} u(x)$ and $f_y(y) = 1$ if

$0 < y \leq 1$, please find the probability density function $f_z(z)$ of the random variable $Z = X + Y$.