1. Solve the differential equation $xy\cos(y^2)y' + 2\sin(y^2) = 0$. (7%)

2. Find the cosine half-range expansion of the function $f(x)$. (7%)

$$f(x) = \begin{cases} \dfrac{2K}{L} & (0 < x < \dfrac{L}{2}) \\ \dfrac{2K}{L}(L - x) & (\dfrac{L}{2} < x < L) \end{cases}$$

3. Find the Fourier cosine integral of $f(x)$. (6%)

$$f(x) = \begin{cases} x & (0 < x < a) \\ 0 & (x > a) \end{cases}$$

4. Solve the differential equation by means of the **Laplace transformation**. (15%)

   $y'' + 2y' + 10y = r(t)$

   where

   $$r(t) = \begin{cases} 1 & (0 < t < \pi) \\ -1 & (\pi < t < 2\pi) \end{cases} \qquad r(t + 2\pi) = r(t).$$

5. Diagonalize the matrix $A = \begin{bmatrix} -1.8 & 0.4 & -0.4 \\ 0 & -1 & 0 \\ -0.4 & 0.2 & -1.2 \end{bmatrix}$. (10%)

6. Using the divergence theorem to evaluate $\int\int_S (7x\mathbf{i} - z\mathbf{k}) \bullet \mathbf{n} \, dA$ over the sphere $S : x^2 + y^2 + z^2 = 4$. (10%)

7. Solve $X' = \begin{bmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix} X$. (15%)

8. z is a complex variable, prove that $\sin z^2 + \cos z^2 = 1$. (5%)

9. Find the images of the circles $|z| = 1$ and $|z| = 2$ under the transformation $T(z) = \dfrac{(z + 2)}{(z - 1)}$, and what are the images of the interiors of these circles? (Please plot your answer and explain it). (15%)

10. Find all values of z satisfying the given equations (a) $\sin z = 2$; and (b) $\cos z = 10$, respectively. (10%)