共 5 題,每題 20 分

1. Andrea has 46 rectangular pieces of paper. If ℓ, w (measured in centimeters) denote the length and width, respectively, of each rectangular piece, then for this situation we find that each of ℓ, w is a positive integer, where $1 \leq w \leq \ell \leq 90$. From among these 46 rectangles, prove that Andrea can select two, say R_1 and R_2, so that R_2 completely covers R_1 when R_2 is placed on top of R_1.

2. Let $A = \{a,b,c,d\}$ and let $R = \{(a,a), (b,b), (c,a), (c,b), (c,c), (c,d),(d,a), (d,d)\}$ be a relation on A
   (A) Verify that (A,R) is a poset and find its Hasse diagram.
   (B) Topologically sort (A,R).
   (C) How many more ordered pairs are needed to extend (A,R) to a totally ordered set?.

3. (A) What is the relation between partition and equivalence relation?
   (B) Let $A = \{a,b,c,d\}$. Find the number of ways in which A can be partitioned into (a) two blocks. (b) three blocks.
   (C) Find the number of equivalence relations on A.

4. Let A be the adjacent matrix of a simple graph G. Explain how to find
   (A) $\deg(v_i)$ if A is given.
   (B) $\deg(v_i)$ if A^2 is given.
   (C) number of triangles in G if A^3 is given.(e.g. K_4 has 4 triangles)
   (D) $\sum_i\sum_j A^2[i,j]$ if $\deg(v_i)$ is given for each i.

5. State whether the argument given below is valid or not. If it is valid, identify the tautology or tautologies on which it is based.
   (A) If New York is a big city, then New York has tall buildings.

   New York has tall buildings.

   Therefore New York is a big city.
   (B) I will become famous or I will not become a writer.

   I will become a writer.

   Therefore I will become famous.
   (C) If I try hard and I have talent, then I will become a musician.

   If I become a musician, then I will be happy.

   Therefore if I will not be happy, then I did not try hard or I do not have

   talent.
   (D) If 2+3=6, then 2+4=8.

   2+3≠6.

   Therefore 2+4≠8.