

國立中央大學八十四學年度碩士班研究生入學試題卷

所別：資訊工程研究所

組

科目：離散數學

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1. Suppose p is an atom. Define a sequence q_0, q_1, q_2, \dots of propositions as follows:

$$q_0 = p \quad \text{and} \quad \text{for } n \geq 1 \quad q_n = (q_{n-1} \rightarrow p)$$

- (a) For which values of n (if any) is q_n a theorem? (5%)
(b) Give a proof for your answer. (10%)

2. (a) For what values of $l, m,$ and n is the complete tripartite graph $K_{l,m,n}$ planar? (5%)
(b) Give a proof for your answer. (10%)

3. Let $S = \{1, 2, \dots, 8\}$ and F be the set of all onto functions from S to S .
(a) How many functions in F satisfying $f(i) \neq i$ for all $i \in S$. (10%)
(b) How many functions in F satisfying $|f(i) - i| \leq 1$ for all $i \in S$. (10%)

4. Four ordinary, 6-sided dice are rolled. Let X denote the sum of the four values showing, and Y denote the maximum of the four values showing.
(a) What is $E(X)$, the expectation of X . (10%)
(b) What is $E(Y)$. (10%)
(c) What is the probability of the event: $X=14$. (10%)

5. Let $x_0 = 0$, and for $n \geq 1$, define

$$x_n = 2x_{n-1} \quad \text{if } n \text{ is even}$$

$$x_n = 2x_{n-1} + 1 \quad \text{if } n \text{ is odd}$$

Find a general formula for x_n . (10%)

6. Let $A = \{1, 2, \dots, n\}$, where n is even. Consider the following claim: If k numbers are selected from A , at least two of them have sum $n+1$. Verify that the claim holds in general with $k > n/2$. (Hint: Use pigeonhole principle.) (10%)