

所別：電機工程學系碩士班 甲、乙組(一般生)(學位在職生) 科目：工程數學  
丙組(一般生)  
丁組

(1) Solve the following differential equation. (15%)

$$y'' - 4y' + 5y = e^{2x} \csc(x).$$

(2) Find the inverse transform of the following function. (15%)

$$H(s) = \ln \left[ 1 - \frac{a^2}{s^2} \right]$$

(3) Find the eigenvalues of

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ where } a_{mn} = \int_{-1}^1 \sin\left(\frac{m\pi}{2}(x+1)\right) \left[ -\frac{d^2}{dx^2} - x \right] \sin\left(\frac{n\pi}{2}(x+1)\right) dx. (20\%)$$

(4) Find the inverse  $A^{-1}$  of

$$A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}. (10\%)$$

(5) Find the principal value of  $(-i)^i$ . (10%)

(6) Find the linear fractional transform that maps  $0, i, -i$  onto  $2i, \infty, 0.5+i$ , respectively. (10%)

(7) Evaluate the Cauchy principal value of  $\int_{-\infty}^{\infty} \frac{\cos(2x)}{(x^2+1)^2} dx$ . (10%)

(8) Using the Fourier integral, show that the given integral represents the indicated function. (Show the details of your work.). (10%)

$$\int_0^{\infty} \frac{\sin(\omega) \cos(x\omega)}{\omega} d\omega = \begin{cases} \pi/2 & \text{if } 0 \leq x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$