國立中央大學九十一學年度碩士班研究生入學試題卷

1. $A=\begin{bmatrix}2&1&1\\2&3&2\\1&1&2\end{bmatrix}$，please compute $A^3-7A^2+11A-4I$ detailedly。
(10%)

2. (a) If λ is an eigenvalue of an invertible matrix A，x is the eigenvector corresponding to λ, prove that $\frac{1}{\lambda}$ and x are the eigenvalue of A^{-1} and its corresponding eigenvector respectively. (5%)
(b) If $A=\begin{bmatrix}0&0&-2\\1&2&1\\1&0&3\end{bmatrix}$, show the result of (a) by using any one eigenvalue of A. (5%)

3. (a) Suppose $[x,\ y,\ z]=x\mathrm{i}+y\mathrm{j}+z\mathrm{k}$ denotes a vector function, where x, y, z are Cartesian coordinates. If we have a function $f(x,y,z)=2x^2+3y^2+z^2$, find its directional derivative at the point P:(2,1,3) in the direction of the vector v=i-2k, and then explain the mathematic meaning of the above result. (10%)
(b) Using the gradient of $f(x,y,z)=2x^2+3y^2+z^2$ to find the divergence of grad f. (5%)

4. Solve the differential equation
$4x^2y''+4xy'-y=\frac{12}{x}$ (10%)

5. Find the inverse transform of the function
$\ln(1+\frac{\omega^2}{s^2})$ (7%)

6. Solve the integral equation
$y(t)=t+\int_0^t \sin(t-\tau)y(\tau)d\tau$ (8%)

7. Find the cosine half-range expansion of the function $f(x)$.
$$f(x)=\begin{cases}\frac{2k}{L}x, & if \quad (0<x<\frac{L}{2})\\ \frac{2k}{L}(L-x), & if \quad (\frac{L}{2}<x<L)\end{cases}$$ (10%)

8. (10%) A complex function $f(z)$ is said to be analytic in a domain D if $f(z)$ is defined and differentiable at all points of D. Find the most general analytic function $f(z)$ whose real part is x^2-y^2-x where $z=x+iy$ with i being the imaginary unit, i.e., $i=\sqrt{-1}$, and x, y both real.

9. (10%) Let $f(z)=(z-z_0)^m$ be a complex function where m is an integer and z_0 a complex constant. Integrate counterclockwise around the circle C of radius ρ with the center at z_0, where $\rho>0$.

10. (10%) Derive the following real integral:
$$\int_0^\infty \frac{1}{1+x^4}dx.$$
(Hint: You may use the residue integration method.)