科目：工程數學D(5006)　　校系所組：中大照明與顯示科技研究所(乙組)
中大電機工程學系(電子組、固態組)
交大電子研究所(甲組、乙組)
交大電控工程研究所(乙組、丙組)

參考用

1. Let $\boldsymbol{A}=\begin{bmatrix}4 & -1 & -1\\ 5 & 3 & 4\\ 2 & 4 & 5\end{bmatrix}$ and $\boldsymbol{B}=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ -1 & 0 & 0\end{bmatrix}$

   (a) Compute $\left(\boldsymbol{A}^T\boldsymbol{B}\right)^T$ and $\boldsymbol{AB}^{15}$. (6%)

   (b) Compute $det(\boldsymbol{A})$ and $det\left(2\boldsymbol{A}^3\right)$. (6%)

   (c) Find $\boldsymbol{A}^{-1}$. (5%)

2. Based on the LU-factorization, a matrix $\boldsymbol{A}$ can be expressed as $\boldsymbol{A}=\boldsymbol{LU}$, where $\boldsymbol{L}$ is a lower triangular matrix with all diagonal entries equal to 1 and $\boldsymbol{U}$ is an upper triangular matrix.

   If $\boldsymbol{A}=\begin{bmatrix}-1 & 1 & 0\\ 1 & 1 & 2\\ -2 & 4 & 3\end{bmatrix}$, what are the matrices $\boldsymbol{L}$ and $\boldsymbol{U}$? (8%)

3. Let the linear transformation be $L:\Re^{2\times 2}\to\Re^{2\times 2}$ be defined by

$$L(A)=\frac{1}{2}\left(A+A^T\right)$$

   for all $A\in\Re^{2\times 2}$.

   (a) Find the matrix representation of L with respect to the standard (ordered) basis

$$\left\{\boldsymbol{E}_1=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix};\boldsymbol{E}_2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix};\boldsymbol{E}_3=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix};\boldsymbol{E}_4=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right\}\ (5\%)$$

   (b) Determine the kernel and range of L. What are their dimensions? (8%)

注意：背面有試題

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4. Let $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$.

   (a) Compute the eigenvalues of A. (3%)

   (b) Compute $f(A) = 2A^3 + A^2 - 5A + 3I$, where I is the 2×2 identity matrix. (4%)

   (c) Determine $\lim_{k\to\infty} A^k \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. (5%)

5. True or false, **MUST with reason or counterexample**.

   (a) Let C be a nonsingular matrix and $Y(x)$ be a fundamental matrix of the linear system $y'(x) = Ay(x)$, where $y(x)$ is a vector function of dimension n. Then $CY(x)$ is also a fundamental matrix. (4%)

   (b) The unique solution of the following initial value problem
   $$y'(x) = y(x)(1 - y(x))(3 - y(x))\sin(y(x)),\quad y(0) = 0.5$$
   is always increasing and between 0 and 1. (4%)

   (c) Let $a, b \in \Re$ and $a>0$. Then all the solutions of $y''(x) + ay'(x) + by(x) = 0$ have the property of $y(x) \to 0$ as $x \to \infty$. (4%)

   (d) If the Wronskian of smooth functions $f_1(x), \cdots, f_n(x)$ vanishes at every point of the real line, then the n functions are linearly dependent. (4%)

6. (a) Please find the complete set of solutions of $y''(x) + y(x) = 0$. (3%)

   (b) Explain why you have found all possible solutions in part (a). (3%)

   (c) Please determine the condition on the complex number c such that the solution of the differential equation $y''(x) + y(x) = e^{cx}$ with $y(0)=1$ is bounded. (3%)

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7. Let $y_1(t)$ and $y_2(t)$ be two solutions of $(t+1)y''-(t+2)y'+y=0$ on the interval $(-1,\infty)$ satisfying $y_1(0)=2$, $y_1'(0)=1$, $y_2(0)=1$ and $y_2'(0)=1$.

Let $W_{[y_1,y_2]}(t) \triangleq \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ be the Wronskian of y_1 and y_2. Suppose that $y_{p_1}(t)$ and $y_{p_2}(t)$ be two particular solutions of the following two initial-value problems

$$(t+1)y''-(t+2)y'+y=t^2;\ y(0)=1,\ y'(0)=0 \text{ and}$$

$$(t+1)y''-(t+2)y'+y=te^t;\ y(0)=0,\ y'(0)=1$$

on the interval $(-1,\infty)$, respectively.

(a) Compute $W_{[y_1,y_2]}(t)$. (4%)

(b) Solve the initial value problem

$$(t+1)y''-(t+2)y'+y=t^2+2te^t;\ y(0)=2,\ y'(0)=1$$

(in terms of y_1, y_2, y_{p1} and y_{p2}) (2%)

8. (a) Solve for $y(t)$ when $\int_0^t \frac{1}{\sqrt{t-\tau}} y(\tau)d\tau = t^2$. Hint: $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. (5%)

(b) The differential equation $f(t)\frac{dy}{dt}+t^2+y=0$ is known to have an integration factor $u(t)=t$. Find all possible functions $f(t)$. (4%)

(c) Find a particular solution in Fourier series of the equation $y'+y=f(t)$, where $f(t)=|t|$, $-\pi \le t < \pi$ and $f(t+2\pi)=f(t)$ $\forall t$. (10%)