# 國立中央大學八十九學年度轉學生入學試題卷

大氣科學系 三年級 科目：應用數學 共 1 頁 第 1 頁

1. What is superposition principle? Does it hold for nonlinear equation? For nonhomogeneous linear equation? For homogeneous linear equation? Why is it important? What do we mean by "resonance"? Where and under conditions does it occur?

(10%)

2. Solve the following initial value problem.

$$y'' + 4y = 8e^{-2x} + 4x^2 + 2, \qquad y(0) = 2, \qquad y'(0) = 2.$$

(10%)

3. Why is orthogonality of function is important? How is it defined? What is a Sturm-Liouville problem? What does it have to do with orthogonality?

(10%)

4. Find the eigenvalues and eigenfunctions of the following problem.

$$y'' + \lambda y = 0, \qquad y(-\pi/2) = y(\pi/2) = 0$$

(10%)

5. Find the eigenvalues and eigenvectors of the following problem.

$$\begin{bmatrix} 4 & -6 & -6 \\ 0 & -2 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$

(10%)

6. Define and state the physical meaning of gradient, divergence and curl. What is Green's theorem? What is divergence theorem? What is Stokes's theorem?

(15%)

7. Find the Fourier transform of f(x) = kx if a < x < b and f(x) = 0 otherwise.

(10%)

8. Why did we sometimes use polar or spherical coordinates? Why and where did Bessel's equation occur? Legendre's equation? What is the method of separating variables? Give an example. In separating the heat equation we got exponential functions. Why? Why not in the case of the wave equation?

(15%)

9. Evaluate the following integral.

$$\int_{-\infty}^{\infty} \frac{dx}{1 + 4x^4}$$

(10%)