

類組：電機類 科目：工程數學 D(3006)

※請在答案卷內作答

- 本試卷共十四題，答錯不倒扣。
- 多選題(一~四)答案可能有一個或多個，請選出所有正確答案。
- 計算題(五、六)必須列明計算過程，否則不予計分。
- 填充題(七~一四)寫出答案即可，不需計算過程。

一、多選題 (5%) Let A be an $m \times n$ real matrix, B be an $n \times m$ real matrix, and $\mathbf{b} \in \mathbb{R}^m$. If $\text{rank} A = m < n$, which of the following equations always have a solution (possibly infinitely many) for any B and \mathbf{b} ?

- (1) $A^T \mathbf{x} = B\mathbf{b}$
- (2) $AA^T \mathbf{x} = \mathbf{b}$
- (3) $AA^T \mathbf{x} = AB\mathbf{b}$
- (4) $A^T A\mathbf{x} = A^T \mathbf{b}$
- (5) $A^T A\mathbf{x} = B\mathbf{b}$

二、多選題 (5%) Let A and B be $n \times n$ real symmetric matrices. Which of the following statements are true?

- (1) If $AB = 0$, then all eigenvalues of BA are zero.
- (2) If $AB = 0$, then $\text{rank}(A) + \text{rank}(B) \leq n$.
- (3) If $\text{rank}(A) = \text{rank}(B)$, then $\text{rank}(A^2) = \text{rank}(B^2)$.
- (4) If $A^2 = 0$, then $A = 0$.
- (5) If $B^2 = I$, then $B = \pm I$.

三、多選題 (5%) Let A and B be $n \times n$ matrices. Which of the following statements are true?

- (1) If A is similar to B , then AB is similar to BA .
- (2) If A is similar to B , then $A^T A$ is similar to $B^T B$.
- (3) If A is similar to B , then A^2 is similar to B^2 .
- (4) If A is nonsingular, then AB is similar to BA .
- (5) It is possible that A is not similar to B , but A^2 is similar to B^2 .

四、多選題 (5%) Let A be an $n \times n$ real matrix. Which of the following statements are true?

- (1) If all the eigenvalues of A are positive, then $\mathbf{x}^T A\mathbf{x} > 0$ for every nonzero $\mathbf{x} \in \mathbb{R}^n$.
- (2) If all the eigenvalues of A are positive, then $\det(A + A^T) > 0$.
- (3) If $\mathbf{x}^T A\mathbf{x} > 0$ for every nonzero $\mathbf{x} \in \mathbb{R}^n$, then $\det(A) > 0$.
- (4) If $\mathbf{x}^T A\mathbf{x} < 0$ for every nonzero $\mathbf{x} \in \mathbb{R}^n$, then $\det(A) < 0$.
- (5) If $\mathbf{x}^T A\mathbf{x} > 0$ for every nonzero $\mathbf{x} \in \mathbb{R}^n$, then $\det(A + A^T) > 0$.

注意：背面有試題

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五、計算題 (15%) Let

$$A = \begin{bmatrix} 3 & 3 & 3 \\ -6 & 6 & 2 \\ 7 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 \\ -6 & 6 & 2 \\ 7 & 1 & 2 \end{bmatrix}^{-1} \quad \text{and } \mathbf{b} = \begin{bmatrix} 15 \\ -14 \\ 25 \end{bmatrix}.$$

- (1) (5%) Find the general solution (also called the complete solution) of $A\mathbf{x} = \mathbf{b}$.
- (2) (5%) Find the reduced row echelon form of A .
- (3) (5%) Find the distance from \mathbf{b} to the row space of A .

六、計算題 (15%) Let

$$A = \frac{1}{2} \begin{bmatrix} 1 & 1 & \sqrt{2} & 0 \\ 1 & 1 & -\sqrt{2} & 0 \\ 1 & -1 & 0 & \sqrt{2} \\ 1 & -1 & 0 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}.$$

Define $\|\mathbf{x}\| = \sqrt{\mathbf{x}^T \mathbf{x}}$.

- (1) (5%) Find $\text{rank} \begin{bmatrix} AA^T & A \\ A^T & A^T A \end{bmatrix}$.
- (2) (5%) Find $\det(I_4 + AA^T)$.
- (3) (5%) Find $\max_{\|\mathbf{x}\|=1} \|A\mathbf{x}\|$.



七、填充題 (5%) The complete solutions of the Bernoulli differential equation $xy' + 6y = 3xy^{4/3}$ are ().

八、填充題 (5%) Consider the linear differential equation $y'' + ay' + by = 0$, where a and b are real constants. Then all the solutions of the equation are bounded no matter what initial state is chosen if the condition () is satisfied.

九、填充題 (5%) The solutions of the nonlinear differential equation $yy'' = (y')^2$ are ().

十、填充題 (5%) The complete solutions of the linear differential equation $4x^2y'' + 8xy' + y = 0$ are ().

十一、填充題 (5%) Suppose that both $x^2e^{-3x} \sin(2x)$ and x^2e^{-5x} are solutions of a linear constant-coefficient homogeneous differential equation. Then the order of this differential equation is at least ().

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十二、填充題 (5%) The solution of $\dot{x} = Ax$ is known to be $x(t) = e^{At}x(0)$. Suppose that

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} \text{ and } e^{At} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}. \text{ Then } a+b+c = (\quad).$$

十三、填充題 (10%) The Laplace transform of $f(t)$ is defined as $L\{f(t)\} \equiv \int_0^{\infty} f(t)e^{-st} dt$ for those s such that the integral exists. Then $L\{te^{5t} \sin t\} = (A)$ for $\text{Re}(s) > (B)$.

十四、填充題 (10%) Let $f(x)$ be a periodic function with period 2 and $f(x) = x^2$, $0 < x \leq 2$, then the Fourier series of $f(x)$ is (A) . Use this Fourier series to find $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = (B)$.

參考用

