國立中央大學103學年度碩士班考試入學試題卷

所別：機械工程學系碩士班 甲組(固力與設計)(一般生)　科目：工程數學　共 2 頁　第 1 頁
機械工程學系碩士班 乙組(製造與材料)(一般生)
機械工程學系碩士班 丙組(熱流)(一般生)
能源工程研究所碩士班 不分組(一般生)
機械工程學系光機電工程碩士班 乙組(光機)(一般生)
本科考試可使用計算器，廠牌、功能不拘　*請在試卷答案卷（卡）內作答

Ordinary Differential Equations

1. Find the solution for the following ordinary differential equations (ODEs):

(a) $y'+y=-x/y$ **(5%)**

(b) $y''+4y'+3y=65\cos(2x)$ **(5%)**

(c) $\begin{cases} y_1'=y_1+y_2+10\cos x \\ y_2'=3y_1-y_2-10\sin x \end{cases}$, find y_1 and y_2 =? **(5%)**

2. For a homogenous ODE given as:

$$x^3y'''-3x^2y''+6xy'-6y=0 \qquad (1)$$

(a) Find three solutions $y_1(x)$, $y_2(x)$, and $y_3(x)$ that can form a basis of solutions, show that they are linear independent, for Eq. (1). **(5%)**

(b) If there is a non-homogenous term $r(x)=x^4\ln x$ of Eq. (1), then Eq. (1) becomes $x^3y'''-3x^2y''+6xy'-6y=x^4\ln x \qquad (2)$

find the particular solution for Eq. (2), $y_p(x)$ = ? **(5%)**

Partial Differential Equations

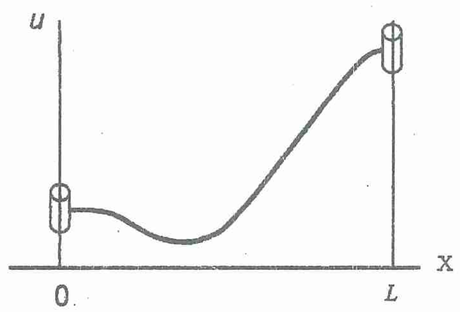
3. Use separation of variables to find the solution. **(10%)**

$$y\frac{\partial^2 u}{\partial x\partial y}+u=0$$

4. A model for the motion of a vibrating string whose ends are allowed to slide on frictionless sleeves attached to the vertical axes ***x=0*** and ***x=L*** is given by the wave equation and the Boundary conditions are:

$$\left.\frac{\partial u}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial u}{\partial x}\right|_{x=L}=0,\ t>0$$

$$u(x,0)=f(x),\quad \left.\frac{\partial u}{\partial t}\right|_{t=0}=g(x),\ 0<x<L.$$



Find the displacement ***u(x,t)*** **(15%)**

注意：背面有試題

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Laplace Transform and Fourier Analysis

5. Solve the integro-differential equation $y'(t)=1-e^{-2t}\int_0^t y(\tau)e^{2\tau}d\tau$, $y(0)=1$ **(10%)**

6. $f(x)=x^2,\ 0<x<2\pi,\ f(x)=f(x+2\pi)$
   (a) Find the Fourier series. **(10%)**
   (b) Evaluate $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}=?$ **(5%)**

Linear Algebra and Vector Analysis

7. Solve the following linear systems

(a) $\begin{bmatrix} 2 & 3 & 1 & -11 \\ 5 & -2 & 5 & -4 \\ 1 & -1 & 3 & -3 \\ 3 & 4 & -7 & 2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}=\begin{bmatrix} 1 \\ 5 \\ 3 \\ -7 \end{bmatrix}$ **(8%)**

(b) $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ **(7%)**

8. Inscribed circles on the sheet metal surfaces are routinely used to investigate the forming limit diagram (FLD) of industrial stamping processes. The deformed sheet stretched from a point P:(x_1,x_2) to Q:(y_1,y_2) can be experimentally measured in a specific direction (principal direction) such that the eigenvalue problems can be applied. The solution procedure typically starts from a boundary circle $x_1^2+x_2^2=1$ and stretches to $\frac{y_1^2}{\lambda_1^2}+\frac{y_2^2}{\lambda_2^2}=1$, where $\lambda_1\lambda_2$ are eigenvalues. The transformation matrix A is designated as $y=Ax$; or $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} 0.5 & 1.5 \\ 1.5 & 0.5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ can be determined experimentally. Please solve eigenvalues and eigenvectors to find the principal directions and indicate the shape of the deformed boundary. **(10%)**

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參考用