國立中央大學103學年度碩士班考試入學試題卷

所別：數學系碩士班 一般組(一般生)　科目：高等微積分　共 1 頁 第 1 頁
數學系碩士班 一般組(在職生)

本科考試禁用計算器　　＊請在試卷答案卷（卡）內作答

1. (i) (10%) If $z = f(x,y)$, $x = r\cos\theta$, and $y = r\sin\theta$, show that

$$(\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2 = (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2.$$

(ii) (10%) If $G(x,y) = f(y+cx) + g(y-cx)$, show that

$$\frac{\partial^2 G}{\partial x^2} = c^2 \frac{\partial^2 G}{\partial y^2}.$$

2. Discuss the convergence and uniform convergence of the following sequence respectively.

(i) (5%) $g_k(x) = \frac{x^k}{k+x^k}$, $x \geq 0$, $k = 1, 2, \cdots$.

(ii) (5%) The sequence $\{T_n\}_{n=1}^{\infty}$ are defined by

$$T_n(x) = \sum_{k=1}^{n} \frac{(\cos kx)^2}{k^2}, \quad x \in (-\infty, \infty).$$

3. Let $\Re$ be the set of all real numbers. Prove or disprove the following statements respectively.

(i) (10%) Let $f : \Re \to \Re$ be a continuous function, and let D be a closed set in $\Re$. Then $f(D)$ is closed.

(ii) (15%) Let $f_k : [0,1] \to \Re$ ba a sequence of continuous functions, and $f_k \to f$ uniformly on $[0,1]$. Then the family $\{f_k\}_{k=1}^{\infty}$ is equicontinuous.

(iii) (10%) Let $f : [a,b] \to \Re$ ba a sequence of Riemann integrable functions, and let f_k converges pointwise to f. Then f is Riemann integrable on $[a,b]$.

4. (i) (10%) Find and classify the critical points of the function

$$f(u,v) = 2uv(12 - 3u - 4v).$$

(ii) (10%) Find the absolute maximum and minimum of

$$f(u,v) = u^2 + v^2 + v \text{ on the disc } u^2 + v^2 \leq 1.$$

5. (15%) Investigate whether the system

$$\begin{cases} x(u,v,w) = u + uvw \\ y(u,v,w) = v + uv \\ z(u,v,w) = w + 2u + 3w^2 \end{cases}$$

can be solved for u, v, w in terms of x, y, z near $(0,0,0)$.

參考用