國立中央大學104學年度碩士班考試入學試題

所別：水文與海洋科學研究所碩士班 不分組(一般生)　科目：微積分　共 1 頁 第 1 頁
水文與海洋科學研究所碩士班 不分組(在職生)

本科考試禁用計算器

＊請在答案卷（卡）內作答

1. Find the following derivatives,

   (a) [10%] Given $y = x^2 \tan\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$

   (b) [10%] Given $\tan(x+y) = x$, find $\frac{dy}{dx}$ in terms of x and y

   (c) [10%] Given $y^2 - 3xy = 1$, find $\frac{d^2y}{dx^2}$ in terms of x and y

2. [15%] Find the Maclaurin series of $\frac{1-e^x}{x}$ to four terms. (Maclaurin series is a Taylor series expansion of a function about 0)

3. Evaluate the following integrations:

   (a) [10%] $\int \frac{x}{1+x^2} dx$

   (b) [10%] $\int e^{2x} \sin x dx$

4. [15%] Show that

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)dx = 0$$

   for any positive integers m and n

5. Given two vectors, $\mathbf{U} = \mathbf{2i} + \mathbf{3j} + \mathbf{k}$, and $\mathbf{V} = \mathbf{2i} + \mathbf{j} + \mathbf{2k}$ find

   (a) [5%] dot product of $\mathbf{U} \cdot \mathbf{V}$

   (b) [5%] cross product of $\mathbf{U} \times \mathbf{V}$

   (c) [10%] the area of triangle formed by $\mathbf{U}$ and $\mathbf{V}$