多重選擇題，每題 5 分，共 50 分。答錯每選項倒扣 1 分

1. If the domain is empty set, which of the following statements are true?
   a) $\forall x P(x)$ is always true regardless what $P(x)$ is.
   b) $\exists x P(x)$ is always true regardless what $P(x)$ is.
   c) $\forall x P(x)$ is always false regardless what $P(x)$ is.
   d) $\exists x P(x)$ is always false regardless what $P(x)$ is.
   e) $\forall x P(x)$ is sometimes true.
2. Given two disjoint sets A and B, which of the following statements are correct?
   a) $A \cap B$ is empty. b) $A \cup B$ is empty. c) $A \setminus B = A$. d) $A \setminus B = B$.
   e) $|A \cup B| = |A| + |B|$.
3. Which of the following sets are considered to be of the same size as the set of nature number N?
   a) The set of Fibonacci numbers $\{F_n\}$. b) The set of odd integers. c) Z.
   d) $Q+$. e) [0.0, 0.1].
4. Which of the following statements about function are correct?
   a) The function $f(x) = (1/x) + 1$ from Q^+ to Q^+ is injective.
   b) The function $f(x) = (1/x) + 1$ from Q^+ to Q^+ is surjective.
   c) The function $f(x) = (1/x) + 1$ from R to R is injective.
   d) The function $f(x) = (1/x) + 1$ from R to R is surjective.
   e) The function $f(x) = x + 1$ from Q^+ to Q^+ is bijective.
5. Which of the following summations are NOT correct?

   a)$\sum_{k=0}^{n} ar^k = \frac{a(r^{n+1}-1)}{r-1}$, where $r \neq 1$. b) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

   c) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. d) $\sum_{k=1}^{n} k^3 = (\frac{n(n+1)}{2})^2$.

   e)$\sum_{k=1}^{\infty} x^k = \frac{1}{1-x}$ for $|x| < 1$.

   For the following two questions, we have the following definitions:
   Let $\boldsymbol{S}$ be a set. Any equivalence relation R_i on $\boldsymbol{S}$ can decide a partition P_i, which is also a set with equivalent classes as its elements. Between every 2 partitions we define a "**derive relation**", *derive*(P_j ,P_k) is true, if and only if every P_j's element (equivalence class) is a subset of some P_k 's element.
6. About **derive relation**, which of the following are correct?
   a) It is also an equivalence relation. b) It is partial order. c) It is a lattice.
   d) Its reflexive closure is itself. e) Its symmetric closure is itself.
7. Consider another relation "**immediate relation**", *immediate*(P_j ,P_k) is true, if and only if ($\forall p_j, p_k$, ($P_j \neq P_k$), and *derive*(P_j ,P_k), and there is no other P_l so that derive(P_j ,P_l) and derive(P_l ,P_k)). Which of the following are collect?

參考用

a) **derive relation**'s transitive closure is itself.

b) **immediate relation**'s transitive closure is **derive relation**.

c) **immediate relation** is total order .

d) **immediate relation** is partial order.

e) **derive relation**'s Hasse diagram is isomorphic to **immediate relation's** underlying undirected graph.

8. Let *P(x)* be the statement "x has a phone", and *C(x,y)* be the statement "x has called y" which can be the correct English interpretation of the logic statement: $\exists x, \forall y, (P(x) \wedge ((x \neq y) \rightarrow \neg C(x,y)))$?

   a) Every one with a phone has called each other except the one without phone.

   b) Someone with a phone has called at least another one.

   c) Exactly one person has a phone and he/she has called everyone else.

   d) Someone has a phone but has not used it to call anyone else.

   e) Not including himself/herself, everyone has commonly not received phone calls from at least one with phone.

9. let $n \in N$, $S = \{1,2,3,\ldots,n\}$ (when n=0, S is empty), and Let a_n denote the number of subsets of S that contain no consecutive integers. What of the following are true?

   a) $a_0 = 0, a_1 = 1$.　b) $a_0 = 1, a_1 = 2$.　c) $a_n = a_{n-1} + n$.　d) $a_n = a_{n-1} + a_{n-2}$.

   e) $a_n = a_{n-1} + 2a_{n-2}$.

10. To solve the recurrence relation: $a_n = 3a_{n-1} + n, n \geq 1, a_0 = 1$ using generating function *f(z)*, what of the following are true?

   a) $f(z) = (\frac{1}{1-3z}) + (\frac{z}{(1-z)^2(1-3z)})$.　b) $f(z) = (\frac{1}{1-3z}) - (\frac{3/4}{(1-z)^2})$.

   c) $f(z) = (\frac{7/4}{1-3z}) - (\frac{1/4}{(1-z)}) - (\frac{1/2}{(1-z)^2})$.　d) $a_n = 3^n - (3/4)(n+1)$.

   e) $a_n = (7/4)3^n - (n/2) - (3/4)$.

複選題，每題 5 分，共 50 分。(答對給 5 分、答錯或不答 0 分)

11. Determine which of the following statements are true?

   (a) Let W be the subspace which consists of vectors of the form (0, b, 0). The dimension of W is 1.

   (b) Let W be the subspace which consists of vectors of the form (a, b, c, d), where d= a−b, c=a−2b. The dimension of W is 2.

(c) All vectors of the form (a, b, c), where b=a mod c, form a subspaces of $\boldsymbol{R}^3$.

(d) All vectors of the form (a, 3, 4) form a subspaces of $\boldsymbol{R}^3$.

12. Suppose that matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$. Determine which of the following statements are true?

(a) The rank of A is at most 2.

(b) A is invertible.

(c) The nullity of A is at most 1.

(d) The nullity of A^T is at most 1.

13. Determine which of the following statements are true?

(a) Let linear operator T be the reflection about the x-axis in R^2. Then T^{-1} is the reflection about the x-axis in R^2

(b) T(x, y)=(2x, y) is a linear operator

(c) T(x, y, z)=(3x−4y, 2x−5z) is a linear transformation.

(d) T: $R^2 \rightarrow R^2$ projects a vector orthogonally onto the y-axis and then reflects that vector about the x-axis. Then the standard matrix of T is $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$.

14. Suppose that matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -7 & 3 \end{bmatrix}$. Which of the following are true?

(a) $A\boldsymbol{x}$=0 has only the trivial solution.

(b) The reduced row-echelon form of A is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $A\boldsymbol{x}=\boldsymbol{b}$ has only the trivial solution for every 3×1 matrix $\boldsymbol{b}$.

(d) The column vectors of matrix A are linearly independent.

15. Let $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$. Determine which of the following statements are true?

(a) $adj(A)=\begin{bmatrix}12 & 4 & 12\\ 6 & 2 & -10\\ -16 & 16 & 16\end{bmatrix}$ (b) $A^{-1}=\frac{1}{64}\begin{bmatrix}12 & 4 & 12\\ 6 & 2 & -10\\ -16 & 16 & 16\end{bmatrix}$

(c) The column vectors of A form a basis for R^3. (d) $\det(A^2)=(\det(A))^2$.

16. $\boldsymbol{A}$ and $\boldsymbol{B}$ are two invertible matrices. If $\boldsymbol{A}$ is similar to $\boldsymbol{B}$, which are correct?
    (a) $\boldsymbol{A}$ and $\boldsymbol{B}$ have the same eigenvectors.
    (b) $\boldsymbol{A}^2$ is similar to $\boldsymbol{B}^2$.
    (c) $\boldsymbol{A}^{\mathrm{T}}$ is similar to $\boldsymbol{B}^{\mathrm{T}}$.
    (d) $\boldsymbol{A}^{-1}$ is similar to $\boldsymbol{B}^{-1}$.
    (e) $\det \boldsymbol{A} = \det \boldsymbol{B}$.

17. If A is diagonalizable and has eigenvalue λ, then
    (a) $\boldsymbol{A}^{\mathrm{T}}$ has eigenvalue λ.
    (b) $\boldsymbol{A}^{-1}$ has eigenvalue $1/\lambda$.
    (c) $\boldsymbol{A}^2$ has eigenvalue λ.
    (d) $\boldsymbol{A}$ is invertible.
    (e) $\boldsymbol{A} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{-1}$, where matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ are unique.

18. If A is a $m\times n$ matrix and A can be $\boldsymbol{QR}$ factorized. $\boldsymbol{W}$ is the set of all columns of $\boldsymbol{A}$, and $\boldsymbol{W}^{\perp}$ is the orthogonal complement of $\boldsymbol{W}$, then
    (a) $\boldsymbol{W}$ is a subspace.
    (b) $\boldsymbol{W}^{\perp}$ is always a subspace.
    (c) $(\boldsymbol{W}^{\perp})^{\perp} = \boldsymbol{W}$.
    (d) $\boldsymbol{W}^{\perp} = \mathrm{Nul}\,\boldsymbol{A}^{\mathrm{T}}$.
    (e) $m \geq n$.

19. Which are correct for matrix $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}$
    (a) If $(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})$ is not invertible, then the inconsistent linear system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ has no least-squares solution.
    (b) If $\boldsymbol{A}$ has orthonormal columns, then $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}$ is an identity matrix.
    (c) If square matrix $\boldsymbol{A}$ has one zero column, then $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}$ is not diagonalizable.
    (d) The eigenvalues of matrix $(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})$ are always arbitrary real number.
    (e) Rank $(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})$ = Rank $\boldsymbol{A}$.

20. If $\boldsymbol{A}$ is not a square matrix, $\boldsymbol{A}$ cannot be factorized into
    (a) $\boldsymbol{A} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{-1}$, where $\boldsymbol{D}$ is a diagonal matrix.
    (b) $\boldsymbol{A} = \boldsymbol{Q}\boldsymbol{R}$, where $\boldsymbol{Q}$ has orthonormal columns.
    (c) $\boldsymbol{A} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{\mathrm{T}}$, where $\boldsymbol{D}$ is a diagonal matrix.
    (d) $\boldsymbol{A} = \lambda_1\boldsymbol{u}_1\boldsymbol{u}_1^{\mathrm{T}} + \lambda_2\boldsymbol{u}_2\boldsymbol{u}_2^{\mathrm{T}} + \dots + \lambda_n\boldsymbol{u}_n\boldsymbol{u}_n^{\mathrm{T}}$, where λ_i and $\boldsymbol{u}_i$ are eigenvalues and eigenvectors of $\boldsymbol{A}$.
    (e) $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{T}}$ (singular value decomposition).