

國立中央大學104學年度碩士班考試入學試題

所別：數學系碩士班 計算數學組(一般生) 科目：微積分 共      頁 第      頁

本科考試禁用計算器

\*請在答案卷(卡)內作答

甲、計算、證明題：共 2 大題，每大題 10 分，共 20 分。須詳細寫出計算及證明過程，否則不予計分。

1. Evaluate the integral

(a)  $\int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} dx$ . (5 分)    (b)  $\int_1^{\infty} \frac{\ln x}{x^2} dx$ . (5 分)

2. Evaluate  $\lim_{x \rightarrow \infty} \left( \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} \right)$ .

參考用

乙、填充題：共 10 題，每題 8 分，共 80 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find  $f'(0)$ , if  $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ . Answer : \_\_\_\_\_

2. Find the limit:  $\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x \tan^{-1} t dt$ . Answer : \_\_\_\_\_

3. Evaluate the integral  $\int_{\pi/4}^{\pi/2} \sqrt{1 + \cos 4x} dx$ . Answer : \_\_\_\_\_

4. Find the limit:  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ . Answer : \_\_\_\_\_

5. What value of  $a$  makes  $f(x) = x^2 + (a/x)$  have a point of inflection at  $x = 1$ ?  
Answer : \_\_\_\_\_

6. Find the area of the region in the plane enclosed by the cardioid  $r = 2(1 + \cos \theta)$ .  
Answer : \_\_\_\_\_

7. Find the direction in which the function  $f(x, y) = x^2y + e^{xy} \sin y$  increase most rapidly at  $P_0(1, 0)$ . Answer : \_\_\_\_\_

8. Find the plane tangent to the surface  $x^2 + y^2 + z^2 = 3$  at  $(1, 1, 1)$ .  
Answer : \_\_\_\_\_

9. Evaluate  $\int_0^2 \int_x^2 2y^2 \sin(xy) dy dx$ . Answer : \_\_\_\_\_

10. Evaluate the line integral  $\int_C -y dx + z dy + 2x dz$ , where  $C$  is the helix  $\mathbf{r}(t) = (\cos t) \mathbf{i} + (\sin t) \mathbf{j} + t \mathbf{k}$ ,  $0 \leq t \leq 2\pi$ . Answer : \_\_\_\_\_