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※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

單選(40%)，每題 5 分

1. An alien spaceship streaks past the football stadium along the direction of play at  $0.6c$  (as measured by the players on the field). A football field is 120 yards long by 55 yards wide. What is the length of the football field according to the alien?  
(A) 120 yards  
(B) 150 yards  
(C) 96 yards  
(D) 189.7 yards  
(E) 75.9 yards.
2. A spaceship whose rest length is 300.0 m has a speed of  $0.80c$  with respect to a certain reference frame. A micrometeorite, also with a speed of  $0.80c$  in this frame, passes the spaceship on an antiparallel track. How long does it take this object to pass the spaceship?  
(A)  $1.110 \mu\text{s}$   
(B)  $1.080 \mu\text{s}$   
(C)  $1.055 \mu\text{s}$   
(D)  $1.025 \mu\text{s}$   
(E)  $1.010 \mu\text{s}$
3. Two identical objects, each of rest mass  $m_0$ , moving with equal but opposite velocities of  $0.60c$  in the laboratory reference frame, collide and stick together. The resulting particle has a rest mass  $M_0$ . Is  $M_0$  equal to  
(A)  $1.5 m_0$   
(B)  $2.0 m_0$   
(C)  $2.5 m_0$   
(D)  $3.0 m_0$   
(E)  $3.5 m_0$ .
4. Which of the following phenomena most clearly demonstrates the wave nature of electrons?  
(A) The photoelectric effect.  
(B) Blackbody radiation  
(C) The Compton effect.  
(D) Diffraction of electrons by crystals.  
(E) Non of these answers.
5. A thermal neutron has a speed  $v$  at temperature  $T = 300 \text{ K}$  and kinetic energy  $m_n v^2/2 = 3kT/2$ . By Heisenberg's Uncertainty principle, what is the estimated kinetic energy (in MeV) of a nucleon bound within a nucleus of radius  $10^{-15} \text{ m}$ ?  
(A) 960.48.  
(B) 763.  
(C) 20.48.  
(D) 197.48.  
(E) 177.

注意：背面有試題

參考用

※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

6. A hydrogen atom is in an excited state with the electron in a p-shell. Which of the following sets of quantum numbers  $n, \ell, m$  is not consistent with this statement?
- (A)  $n, \ell, m = 1, 1, 0$ .  
 (B)  $n, \ell, m = 2, 1, 1$ .  
 (C)  $n, \ell, m = 2, 1, 0$ .  
 (D)  $n, \ell, m = 2, 1, -1$ .  
 (E)  $n, \ell, m = 3, 1, 0$ .
7. Consider the electrons in a 1D metal and describe them as free particles. Let  $E(k) = p^2/2m$  be the energy of an electron with momentum  $p$ . Then, the corresponding phase velocity of the electron wave is given by
- (A)  $p/m$   
 (B)  $p/(2m)$   
 (C)  $2p/m$   
 (D)  $2^{1/2}p/m$   
 (E) None of the above
8. Consider two free electrons in 1D having the same wave vector  $k$  but opposite spins, e.g., one with the z-spin being '+' (up) and the other with the z-spin being '-' (down). Let their position coordinates be  $x$  and  $x'$ , and the z-spin variables be  $s$  and  $s'$ , respectively. (We use, for example, the notation  $|s=+\rangle$  to denote the up-spin state of the 1<sup>st</sup> electron, and so on.) Which of the following expressions gives the correct, normalized wave function for the two electrons?
- (A)  $e^{ikx}e^{ikx'}|s=+\rangle|s'=-\rangle$   
 (B)  $e^{ikx}e^{ikx'}(|s=+\rangle|s'=-\rangle + |s=-\rangle|s'=+\rangle)/2^{1/2}$   
 (C)  $e^{ikx}e^{ikx'}(|s=+\rangle|s'=+\rangle + |s=-\rangle|s'=-\rangle)/2^{1/2}$   
 (D)  $e^{ikx}e^{ikx'}(|s=+\rangle|s'=-\rangle - |s=-\rangle|s'=+\rangle)/2^{1/2}$   
 (E) None of the above

複選 (40%)，每題 5 分

9. Which are the following statements correct?
- (A) The maximum kinetic energy of the recoil electron in Compton scattering is given by  $\frac{hf}{\frac{mc^2}{2hf} + 1}$ , where  $f$  is the frequency of the incident photon,  $h$  the Plank constant,  $c$  speed of the light and  $m$  rest mass of an electron.
- (B) The black body radiation curve for different temperatures peaks at a wavelength inversely proportional to the temperature.
- (C) The photon is massless so that the gravitational force will not affect its frequency as it leaves a star.
- (D) The maximum kinetic energy of the emitted photoelectron depends on the frequency of the incident light.
- (E) x-rays were found to be unaffected by electric and magnetic fields, to pass readily through opaque materials and to cause phosphorescent to glow.

注意：背面有試題

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※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

10. X-rays having energy of 250 keV undergo Compton scattering from a target. The scattered rays are detected at  $41^\circ$  relative to the incident rays. Which of the following statements are true: (hint  $\sin 41^\circ = 0.656$ ,  $\cos 41^\circ = 0.7547$ )
- (A) The wavelength of incident X-ray is  $4.96 \times 10^{-3}$  nm.
  - (B) The Compton wavelength shift at this angle is  $5.15 \times 10^{-4}$  nm.
  - (C) The wavelength of scattered X-ray is  $5.95 \times 10^{-3}$  nm.
  - (D) The energy of the scattered X-ray is 223.2 keV.
  - (E) The kinetic energy of the recoiling electron is 26.8 keV.
11. Which are the following statements correct?
- (A) At every instant the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an EM wave equals to the speed of light.
  - (B) The instant energy density associated with the magnetic field of an EM wave equals to the instant density associated with the electric field.
  - (C) The intensity of an EM wave is proportional to the cube of the amplitude of magnetic field.
  - (D) Classical wave equation is invariant under Galileo transformation.
  - (E) The intensity of an EM wave equals the average energy density multiplied by the speed of light.
12. Ytterbium (Yb) has a work function  $\phi = 2.6$  eV. Which of the following statements are true:
- (A) The lowest frequency  $f$  that can liberate an electron from Yb is  $6.29 \times 10^{13}$  Hz.
  - (B) The longest wavelength  $\lambda$  that can liberate an electron from Yb is 476.9 nm.
  - (C) Suppose light with wavelength  $\lambda = 257$  nm impacts some Yb. The voltage  $V_{\max}$  that the liberated electrons from Yb can overcome is 2.23 V.
  - (D) When an unknown light is shone on Yb, it is found that the electrons can only overcome a voltage of  $V_{\max} = 1.75$  V. The wavelength of light is 708.6 nm
  - (E) None of the above
13. Which of the following are true for photons?
- (A) Photons are Bose particles.
  - (B) Two photons are not allowed to occupy the same quantum state.
  - (C) They obey Bose-Einstein distribution with a finite chemical potential.
  - (D) Inside a cavity, the average photon number in a specific cavity mode (with frequency  $f$ ) is approximately given by  $k_B T / (hf)$ ,  $h =$  Planck constant, when  $k_B T / h \gg f$ .
  - (E) None of the above.
14. Which of the following are true for electrons?
- (A) The same quantum state cannot be occupied by more than one electron.
  - (B) Two electrons can have the same spin state.
  - (C) At any temperature, electrons in a solid cannot be found above the Fermi level  $E_F$ .
  - (D) The existence of energy gap in a semiconductor can be proved within quantum physics.
  - (E) None of the above.

注意：背面有試題

參考用

※選擇題請在答案卡內作答，非選擇題請在答案卷內作答

15. Consider the Schrödinger Equation for the time-dependent state vector  $\psi(x,t)$  of a system:

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = H\psi(x,t). \text{ Assume we can find a function } \varphi_E(x) \text{ such that it is an eigenstate}$$

to the Hamiltonian  $H$ :  $H\varphi_E(x) = E\varphi_E(x)$ . Which of the following statements are true?

- (A) A complete solution of the full Schrödinger Equation above is given by the time-independent function  $\psi(x,t) = \varphi_E(x)$ .
- (B) A complete solution of the full Schrödinger Equation above is given by the time-dependent function  $\psi(x,t) = \varphi_E(x) \exp(i\frac{E}{\hbar}t)$ .
- (C) The full solution to the Schrödinger Equation can be written as a product  $c(t)\varphi_E(x)$ , where  $c(t)$  is some complex function of time.
- (D) The function  $c(t)$  above can have any arbitrary functional form.
- (E) If measured at some future time, the energy of the system described by  $c(t)\varphi_E(x)$  is  $E$ .

16. Which of the following statements are correct?

- (A) If electrons have been accelerated from rest through a potential difference of 54 V, then the wavelength of electrons is 1.667 nm.
- (B) For high principle quantum number ( $n$ ) for hydrogen atom, the spacing between the neighboring energy levels is proportional to  $1/n^3$ .
- (C) A particle of mass  $m_e$  trapped in an infinite depth well of width  $L = 1$  nm. Consider the transition from the excited state  $n = 2$  to the ground state  $n = 1$ . The wavelength of light emitted is 886.4 nm.
- (D) The relation for total energy ( $E$ ) and momentum ( $p$ ) for a relativistic particle is  $E^2 = c^2 p^2 + m^2 c^4$ , where  $m$  is the rest mass and  $c$  is the velocity of light. According to the relativistic relations for  $E$  and  $p$ , the product of group velocity and the phase velocity is equal to  $c^2$ .
- (E) If the electron wave function is  $Ce^{ikx}$  between  $x = 2$  and 22 cm, and zero everywhere else, where  $C$  is a real constant, the probability of finding the electron between  $x = 0$  and 4 cm is 0.1.

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參考用

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計算(20%)

1. (10%) For ionic solids, the attraction electrostatic potential energy is

$$U_{\text{attractive}} = -\alpha \cdot k \cdot \frac{e^2}{r}$$

,and the repulsive electrostatic potential energy is

$$U_{\text{repulsive}} = +\frac{B}{r^m}$$

,and the total potential energy is

$$U_{\text{total}} = U_{\text{attractive}} = -\alpha \cdot k \cdot \frac{e^2}{r} + \frac{B}{r^m}$$

- (A) Plot the all three potential energy  $U$  per ion pair versus separation distance  $r$ , and indicate the equilibrium distance  $r_0$  and equilibrium potential energy  $U_0$ . (5%)
- (B) Determinate the equilibrium (minimum) potential energy  $U_0$ . (5%)

2. (10%) For semiconductor, use the conduction band ( $E_c$ ), valence band ( $E_v$ ), energy band gap ( $E_g$ ) electron, hole, photon energy and energy conversation law to

- (A) Illustrate (畫圖說明) the operation principle of light-emitter diode (LED). (5%)
- (B) Illustrate (畫圖說明) the operation principle of solar cell (SC). (5%)