所別： 電機工程學系 碩士班 固態組(一般生)	共2頁 第1頁
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科目： 工程數學

本科考試禁用計算器 ＊請在答案卷 (卡) 內作答

一、 **(10％)** There is a transformation $T(x_1, x_2)=(3x_1+x_2, 2x_1-5x_2)$.

(1) **(5％)** Find the matrix of T relative to the bases $B=\{(1,2),(-1,-2)\}$ and

$B'=\{(2,-1),(-1,2)\}$.

(2) **(5％)** Find $[T(v)]_{B'}$, where $v=\begin{bmatrix} 6 \\ 3 \end{bmatrix}_B$. Note: $v=\begin{bmatrix} 6 \\ 3 \end{bmatrix}_B$ means the vector v is on the basis B.

二、 **(10％)** Please prove that S can be a basis of a matrix $M_{2\times2}$, where

$$S=\left\{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix},\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix},\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix},\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}\right\}.$$

三、 (1) **(5％)** Find the least squares solution of the following system $Ax=b$, where

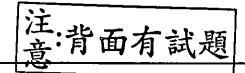$$A=\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \ b=\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

(2) **(5％)** From (1), find the orthogonal projection of b on the column space of A.

四、 **(15％)** Solve the following differential equation (Show the details of your work)

$$3y^2y'+3x^2y^3=e^{-x^3}\cosh x.$$

五、 **(15％)** Find the Laplace transform of the following function (Show the details of your work):

$$f(t)=\frac{k}{p}t \quad \text{if } 0<t<p, \quad f(t+p)=f(t) \text{ and } k \text{ is a constant.}$$

六、 **(10 ％)** Find the harmonic conjugate of $u(x, y)=x^3-3xy^2+5$.

注意：背面有試題

七、(1) **(10％)** For the function $f(z) = \dfrac{1}{\sin z}$, find the first three non-zero

terms of its Laurent expansion about $z = 0$.

(2) **(10％)** Evaluate $\displaystyle\oint_C \frac{1}{z^2 \sin z}\,dz$, where C is the circle $\left| z - \dfrac{1}{2} \right| = 1$.

八、(1) **(3％)** Find the continuous-time non-periodic signal $x(t)$ with its Fourier transform

$X(j\omega) = \dfrac{1}{j\omega + 500}$.

(2) **(7％)** Find the continuous-time non-periodic signal $x(t)$ with its Fourier transform

$X(j\omega) = \dfrac{5(j\omega) - 100}{(j\omega)^2 + 100(j\omega) - 120000}$.

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