

※請在答案卷內作答

1. (24%) Consider the following closed-loop feedback system as depicted in Figure 1, where G_p and G_c denote the system plant and controller, respectively.

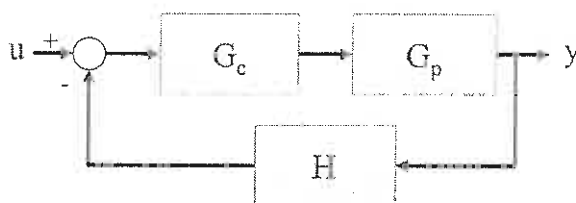


Figure 1.

Let the Laplace transform of the error $e(t)$ be defined as $E(s) = U(s) - Y(s) \cdot H(s)$. Here, $U(s)$, $Y(s)$ and $H(s)$, respectively, denote the Laplace transform of the input, output and feedback block.

- (a) (6%) Let $G_p = \frac{100}{s(s+2)}$, $G_c = 1$ and $H(s) = 0$. Solve the maximum overshoot of the system output $y(t)$ and the corresponding time for the unit-step input, i.e., $U(s) = \frac{1}{s}$.

- (b) (4%) Let $G_p = \frac{100}{s(s+2)}$, $G_c = K$ and $H(s) = 1$. Find the value of K so that the closed-loop poles will have damping ratio $\zeta = 0.5$. In addition, what will be the value of the corresponding undamped natural frequency ω_n ?

- (c) (8%) Let $G_p = \frac{100}{s(s+2)}$, $H(s) = 1$ and the input u be a unit step. Design a PD-controller G_c to make the settling time be less than 0.1 second with damping ratio $\zeta = 0.8$. What will be the corresponding steady-state error of the closed-loop system when the input u becomes unit ramp?

- (d) (6%) Let $G_p = \frac{100}{s(s+2)}$, $G_c = K$ and $H(s) = \frac{s+15}{s+10}$. Find the range of K so that the closed-loop system is stable.

參考用

2. (26%) Consider the closed-loop feedback system as given in Figure 1 above.

- (a) (8%) Let the characteristic equation of the closed-loop system be given by $s^3 + (15 + K)s^2 + (50 + 2K)s + K = 0$. Let $G_c = K$ and $H(s) = 1$. Then find function G_p and plot the root loci for $K \geq 0$.

- (b) (8%) Let the system plant G_p be the same as the one obtained in part (a) and $G_c = K$. Now, let the feedback block be changed from $H(s) = 1$ to $H(s) = \frac{1}{(s+1)(s+0.5)}$. Plot the root loci for $K \geq 0$ and find the range of K for guaranteeing the stability of the closed-loop system.

- (c) (10%) Let the system plant G_p be the same as the one obtained in part (a) and

※請在答案卷內作答

$H(s) = \frac{1}{(s+1)(s+0.5)}$. Now, we change the controller from $G_c = K$ to $G_c = K(s+4)$.

Plot the root loci for $K \geq 0$ and find the range of K for guaranteeing the stability of the closed-loop system.

3. (24%) A control engineer has the experimental data for a minimum-phase open-loop system $G(s)$ and sketches them as the following Nyquist plot.

(a) (8%) Determine $G(s)$ as minimum order as possible.

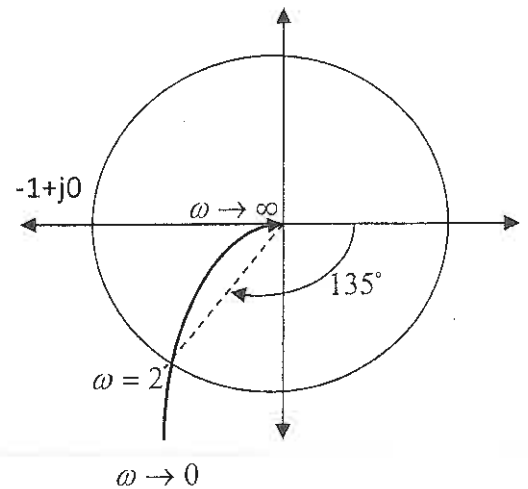
(b) (2%) If a delay e^{-sT} is added to $G(s)$, what is the maximum T so that the system is still stable?

(c) (4%) If a PD controller $G_c(s) = K_p + K_d s$ is added, what the parameters are selected so that the steady state error for unit-ramp input is less than 1% and the maximum overshoot is less than 5%?

(d) (10%) What is the phase margin? If we would like to increase the phase margin by pole-zero cancellation

$G_c(s) = K \frac{s+z}{s+p}$, choose suitable z and plot K v.s. p to

meet the spec. that the steady state error for unit-ramp input is less than 1% and phase margin $\geq 60^\circ$ for $\omega_g = 2$.



4. (26%) The open loop transfer function of a linear time-invariant system is

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{s + \alpha}{s^3 + 7s^2 + 14s + 8}, \text{ and the state equations are } \begin{cases} \dot{x}(t) = Ax(t) + Bu(t); \\ y(t) = Cx(t) + Du(t). \end{cases}$$

(a) (6%) Write the controllability canonical form (CCF) of the state equation.

(b) (3%) Determine all possible α so that the system is either uncontrollable or unobservable.

(c) (6%) Design the control law $u(t) = -Kx(t)$, where $K = [k_1, k_2, k_3]$ and $\alpha = 4$ so that the closed-loop system poles contain $-1 \pm j$.

(d) (5%) Show that the response of the state is $x(t) = \phi(t)x(0) + \int_0^t \phi(t-\tau)Bu(\tau)d\tau$,

where $\phi(t)$ is the state transition matrix and initial condition is $x(0)$.

(e) (6%) If $A = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $C = [1 \ 0 \ 0]$ and $D=1$. Find the state transition

matrix $\phi(t)$ and the transfer function.

注意：背面有試題

參考用