國立中央大學 106 學年度碩士班考試入學試題

所別： 電機工程學系 碩士班 固態組(一般生)
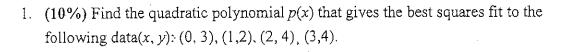　　　 電機工程學系 碩士班 系統與生醫組(一般生)
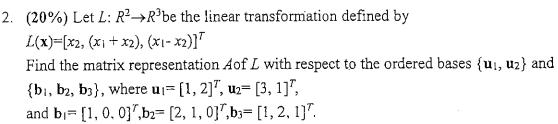
科目： 工程數學

本科考試禁用計算器

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＊請在答案卷　　內作答

參考用

須有計算過程

1. **(10%)** Find the quadratic polynomial $p(x)$ that gives the best squares fit to the following data(x, y): $(0, 3)$, $(1,2)$, $(2, 4)$, $(3,4)$.

2. **(20%)** Let $L: R^2 \to R^3$ be the linear transformation defined by
$L(\mathbf{x})=[x_2, (x_1 + x_2), (x_1 - x_2)]^T$
Find the matrix representation A of L with respect to the ordered bases $\{\mathbf{u}_1, \mathbf{u}_2\}$ and $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, where $\mathbf{u}_1 = [1, 2]^T$, $\mathbf{u}_2 = [3, 1]^T$,
and $\mathbf{b}_1 = [1, 0, 0]^T$, $\mathbf{b}_2 = [2, 1, 0]^T$, $\mathbf{b}_3 = [1, 2, 1]^T$.

3. **(15%)** Find the **Laplace transform** of the following function:
$$f(t) = k \sin\left(\frac{t}{2}\right) \quad (0 < t < 2\pi)$$

4. **(15%)** Solve the following differential equation:
$$y' - 1 = e^{-y} \sin x$$

5. **(10%)** Find all values of i^{3i}.

6. **(10%) (a)** Develop $f(z) = \left(z - \dfrac{\pi}{4}\right)^{-3} \cos z$ into its Laurent series with center

$z_0 = \dfrac{\pi}{4}$ and state its region of convergence.

(10%) (b) Evaluate $\oint_C \left(z - \dfrac{\pi}{4}\right)^{-3} \cos z \, dz$, where C is the ellipse

$x^2 + 16y^2 = 16$ on the z-plane oriented counterclockwise.

7. **(10%)** For a continuous-time non-periodic signal x(t), the signals is
$$x(t) = t \cdot e^{j\omega_0 t} \cdot e^{-at} u(t).$$

Please find its Fourier transform $X(j\omega)$.